

AN APPLICATION OF NONLINEAR FILTERING
THEORY TO PASSIVE TARGET LOCATION
AND TRACKING.

George William Mitschang

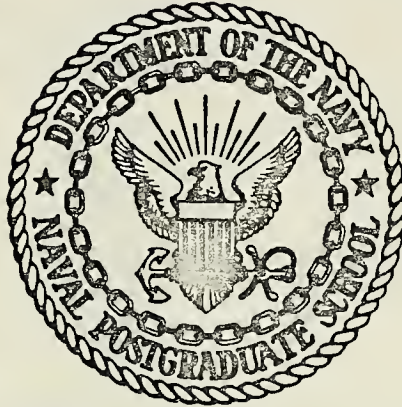
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THESIS

AN APPLICATION OF NONLINEAR FILTERING THEORY
TO PASSIVE TARGET LOCATION AND TRACKING

by

George William Mitschang

June 1974

Thesis Advisor:

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An Application of Nonlinear Filtering Theory
to Passive Target Location and Tracking

by

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LIST OF SYMBOLS FOR TEXT

A	angular rotation parameter for $R_{cpa} - X_{cpa}$ filter - equals +1 for counter-clockwise; equals -1 for clockwise rotation of target about sensor.
d	range of second sensor to first sensor
$d\mathbf{x}(t)$	increment of continuous process $\mathbf{x}(t)$
δ	small vector quantity
δ_{kj}	Kronika delta; = 1 if $k = j$; = 0 if $k \neq j$
$E[\mathbf{w}(k)]$	expected value of the vector, $\mathbf{w}(k)$
$e(j)$	estimation error
F_{AVE}	weighted average of received doppler shifted frequencies
$F_0(k)$	rest or zero-doppler frequency of target
F_{OI}	calculated initial target rest-frequency
Δf	frequency difference
$f(k)$	target doppler shifted frequency measured at time $t(k)$
$\mathbf{f}(\mathbf{x}(k), k), \mathbf{g}(\mathbf{x}(k), k)$	discrete state model terms
$\mathbf{f}(\mathbf{x}(t), t), \mathbf{g}(\mathbf{x}(t), t)$	continuous state model terms
$f_\ell \quad \ell = 1, 5$	X-Y state forcing functions
f_2, θ_2	observations at second sensor
$G(k)$	Kalman gain matrix
$\mathbf{g}^{-1}(\mathbf{x}, (k-1), k-1)$	inverse function of $\mathbf{g}(\mathbf{x}(k-1), k-1)$
$g_\ell \quad \ell = 1, 5$	$R_{cpa} - X_{cpa}$ state forcing functions
$\Gamma(k)$	state forcing matrix at time $t(k)$
$H(k)$	observation matrix at time $t(k)$

$L(e(j))$	cost function
$P(k)$	covariance of error matrix at time $t(k)$
$P'(k)$	predicted covariance of error matrix at time $t(k)$
$p(\underline{x}(0))$	probability density function of the vector random variable $\underline{x}(0)$
$\int_{\underline{x}(k-1)} p(\underline{x}(k), \underline{x}(k-1)) / Z^k d\underline{x}(k-1)$	integration over the probability space containing all permissible values of $\underline{x}(k-1)$
$p(\underline{x}(k+1)/\underline{x}(k))$	conditional probability density function of the vector random variable $\underline{x}(k+1)$ given the vector $\underline{x}(k)$
ϕ	bearing angle of second sensor from first sensor
$\Phi(k)$	state transition matrix at time $t(k)$
$Q(k)$	state excitation covariance matrix at time $t(k)$
$R(k)$	measurement noise covariance matrix at time $t(k)$
R	matrix of state correlation coefficients
$\hat{R}(k)$	range to the target calculated from filtered state estimates at time $t(k)$
$R'(k)$	predict range to the target at time $t(k)$
$R_{cpa}(k)$	range from the sensor to target at closest point of approach (cpa); will be a constant for constant-speed, constant-heading target
R_{cpa_2}	range of target from second sensor at cpa of second sensor
R_{cpaI}, X_{cpaI}	calculation initial position states for $R_{cpa} - X_{cpa}$ filter
R_0	calculated initial target range
r_{xy}	correlation coefficient between sample statistics \bar{x} and \bar{y}
Σ	diagonal matrix of state standard deviations

$\sigma_{\Delta\theta}$	standard deviation of angle difference measurement
$\sigma_{\Delta f}$	standard deviation of frequency difference measurement
$\sigma_{v_{SI}}$	estimate of velocity guess
$\sigma_{\dot{F}_O}$	standard deviation of random rest frequency rate
σ_f^2	variance of frequency measurement noise
$\sigma_{R_{cpaI}}(\Delta\theta)$	equivalent "standard deviation" of R_{cpaI} due to a one-standard-deviation change in $\Delta\theta$
σ_θ^2	variance of angle measurement noise
$\sigma_{\dot{\theta}_s}$	standard deviation of random heading rate
$\sigma_{\dot{v}_s}$	standard deviation of random velocity rate
$\sigma_x^{..2}$	variance of x component acceleration
$\sigma_{xy}^{....}$	covariance of x and y component acceleration
$\sigma_y^{..2}$	variance of y component acceleration
σ_x^2, σ_y^2	sample Monte Carlo variances
$\sigma_x'^2, \sigma_y'^2$	new Monte Carlo variances in rotated system
$\theta(k)$	noisy target bearing at time $t(k)$
$\hat{\theta}(k)$	target bearing angle calculated from filtered state estimates
θ_{AVE}	weight average of initial noisy bearing measurements
θ_{2AVE}	weighted average of calculated bearing angles from second sensor
$\theta_s(k)$	target heading
θ_{sDWN}	solution to initial heading calculation which is down doppler
θ_{sUP}	solution to initial heading calculation which is up doppler
θ_{sI}	calculated initial target heading
$\Delta\theta$	angle difference

Δt	time difference
$\underline{v}(k)$	measurement noise vector
$v_f(k)$	frequency measurement noise
$v_\theta(k)$	angle measurement noise
v_p	velocity of signal propagation in medium
$v_s(k)$	target velocity
v_{sI}	assumed initial target velocity
$v_x(k), v_y(k)$	X and Y components of velocity at time $t(k)$
$\underline{w}(k)$	random system forcing noise
$\tilde{\underline{x}}$	state error vector
$\hat{\underline{x}}(j)$	filter estimates at time $t(j)$ after the measurement at $t(j)$
$\underline{x}'(j)$	predicted state values at time $t(j)$
$\underline{x}(t)$	continuous vector process
$\underline{x}(k)$	vector states
$x(k), y(k)$	X and Y position of target
x', y'	rotated coordinate axes to obtain uncorrelated position statistics
$\bar{x}(k), \bar{y}(k)$	Monte Carlo averages of X and Y target position at time $t(k)$
$x_{\ell 0}$ $\ell=1-5$	calculated initial state values
x_B, y_B	X, Y position of additional sensor
x_B', y_B'	X, Y distance of target to additional sensor
$X_{cpa}(k)$	range of target to cpa at time $t(k)$ - negative before cpa - positive after cpa
X_{cpa2}	range of target to cpa of second sensor
$\underline{z}(t)$	continuous vector observation
$\underline{z}(k)$	discrete observation vector at time $t(k)$

z^k set of all observations from $t(1) - t(k)$
 z^t set of all measurements in interval $[0, t)$
 $\underline{z}'(k)$ predicted measurement vector at time $t(k)$

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I. INTRODUCTION

There are many engineering problems in nonlinear filtering which do not have the luxury of many observations over an extended period of time. The estimator or filter is required to provide a rapid, accurate solution before steady state conditions of the gains and covariance of error are reached. In these situations it is imperative to use as much information as possible from the observations, the known apriori conditions, and the statistical relationship of the initial filter states.

The location of acoustic emitters by remote passive sensors is such a problem. The received acoustic signals may be processed to obtain noisy angle of arrival and noisy doppler shifted frequency data. These, in turn, can be used as observables to provide location and tracking information on the emitting target. The ability to accomplish this with a single sensor (sonobuoy) as well as the flexibility to incorporate additional sensors as available would be highly desirable for the Anti-Submarine Warfare Community. A filter using angle of arrival and doppler-shifted frequency can be applied to other tracking problems in the radar domain as well.

There are presently available methods which use only the doppler-shifted frequency from some array of three or more sensors [46]. In general, they require a good initial

guess as to where the emitter is located because they rely on a linearization method closely resembling a least-square fit of the received frequency data. Currently, research is going on to incorporate angle and frequency data in a common arrangement [35].

A first objective of this research was to apply non-linear filtering theory to the single sensor location and tracking problem and develop an algorithm to solve this estimation problem. The calculation of initial filter states using a single sensor with angle and frequency measurements only, represents a difficult task. A single sensor making either measurement alone does not provide a unique solution and it is only when the combination is used that a single determination of the target position is possible. The geometry of the problem, while simple, does not make determination of initial position using the specified observations very simple. Yet, this initial position estimate must be chosen so that the subsequent linearized filter techniques will converge. In doing so, the effect of various choices which are available to the filter designer became evident. The primary choice is that of a system model, and even for a simple constant-velocity target, differences in the model formulation are possible. The value of the initial filter states and their resultant uncertainty, as reflected in the initial covariance matrix, are seen to play important roles in the time it takes for the filter

to converge. Using all of the covariance terms in this matrix, as opposed to only the diagonal variances, results in better filter performance in the transient phase and lower error variances for a specified track duration. The geometric placement of sensors was considered to determine where they might be placed for best filter performance. It is also important to determine how well the filter is performing while in operation, not so much as a quantitative measure of accuracy, but whether the filter is indeed estimating a reasonable track. Following the lead of Periot [34], the filter residuals were found to be a good measure of a valid track.

In Chapter Two general filtering theory is considered with the emphasis on the underlying probability structure of the optimal filter. This approach clearly points out the simplicity of a linear-Gaussian system and its resultant filtering solution as well as the difficulties and complexities of the nonlinear problem. The continuous formulation of the filtering problem is also reviewed with reference to stochastic integrals and the determination of the strictly formal solution. In the final section of Chapter Two, the techniques that various authors have used to approximate the optimal nonlinear solution will be presented. The approaches range from a direct calculation of the a posteriori density function [10], to a simple linearization of state and observation equations about some nominal state trajectory [38]

Chapter Three describes in detail the single sensor passive location and tracking problem and will develop the two state formulations. The equations for an Extended Kalman Filter for each case are presented. The important initial condition equations are derived and the initial covariance of error expressions for the states are found. The technique used to obtain the initial covariance matrix is new and consists of a direct calculation of the changes in the nonlinear initialization equations when the input measurements change by one standard deviation. The values for the filter state excitation matrix (which turn out to be state dependent), are presented next. The ability to add additional sensors or observables to the filter is an important advantage of the Extended Kalman method and this is developed next.

Chapter Four contains a new approach to the determination of error covariances for nonlinear functions of random variables. This approach is an extension of the method used to obtain the initial covariance of state error matrix. The method may be readily applied to formulate the complete nonlinear filtering problem using the linear Kalman filter equations. The one step prediction and updating is accomplished by a direct calculation of the change in the nonlinear state or observation function caused by a one-standard-deviation change in each state variable. The matrix of the correlation coefficients of the states is found to play an important role.

A Monte Carlo simulation of the two filters developed in Chapter Three is given in Chapter Five for a wide range of problem geometry. The effects of initial conditions and initial covariance matrices will be seen to play an important role in performance of the filter.

The final chapter summarizes the results of this investigation and presents the conclusions and suggestions for further study.

II. FILTERING THEORY

A. THE PROBABILISTIC APPROACH FOR THE DISCRETE CASE

Since the extraction of states or parameters from noisy measurements is fundamentally a problem of estimating random variables, it is important to consider the underlying probability theory of a general filtering problem. Indeed the characterization of the estimates with the appropriate probability density function is the most complete knowledge that can be obtained about these estimates. The evolution of this density function in time, and its change with incoming measurements, represents the general solution to the estimation problem. How the value for the estimate is determined will be governed by the cost or penalty charged for making a wrong choice [31,44].

Assume a discrete system of the following form

$$\underline{x}(k+1) = \underline{f}(\underline{x}(k),k) + \underline{g}(\underline{x}(k),k) \cdot \underline{w}(k) \quad (2.1)$$

$$\underline{z}(k) = \underline{h}(\underline{x}(k),k) + \underline{v}(k) \quad (2.2)$$

with \underline{x} the vector of states or parameters (or both) to be estimated and \underline{z} the vector of observations. The notation used throughout the text is that vector quantities are lower case letters and underlined and matrices are given by upper case letters. The initial states $\underline{x}(0)$ are assumed to be from some initial density $p(\underline{x}(0))$ and the statistics of the system and measurement noise \underline{w} and \underline{v} are known and given by

$$E[\underline{w}(k)] = 0$$

$$E[\underline{v}(k)] = 0$$

$$E[\underline{w}(k) \cdot \underline{v}(j)] = 0 \quad \text{for all } k, j$$

$$E[\underline{g}(\underline{x}(k), k) \cdot \underline{w}(k) \cdot (\underline{g}(\underline{x}(k), k) \cdot \underline{w}(k))^T] = Q(k) \delta_{kj}$$

$$E[\underline{v}(k) \cdot \underline{v}^T(j)] = R(k) \cdot \delta_{kj}$$

with

$$\begin{aligned} \delta_{kj} &= 1 & k=j \\ &= 0 & k \neq j \end{aligned} ,$$

and

$$E[\underline{w}(k) \cdot \underline{x}^T(0)] = 0$$

$$E[\underline{v}(k) \cdot \underline{x}^T(0)] = 0 \quad . \quad (2.3)$$

Note that the above system of equations defines a Markov process; that is, a process whose probability law for future times depends only on the present state values. In terms of the density functions

$$p(\underline{x}(k+1)/\underline{x}(k), \underline{x}(k-1) \dots \underline{x}(0)) = p(\underline{x}(k+1)/\underline{x}(k)) \quad (2.4)$$

The Markov property of a process is very fundamental and along with the uncorrelated noise assumptions written above, allows great simplification in the recursive expressions developed later.

Let

$$Z^k = \{\underline{z}(k), \underline{z}(k-1), \dots, \underline{z}(1)\} \quad (2.5)$$

be the set of all measurements on the system up to the present time. The probabilistic knowledge one has about the system, given the set of measurements Z^k , is contained in the conditional probability density function of the states given

the measurements: $p(\underline{x}(j)/Z^k)$. Note that the index value j and superscript k are not necessarily the same. This allows the three main classes of problems – smoothing for $j < k$, filtering for $j = k$, and prediction for $j > k$ – to be considered simultaneously.

1. The Cost Function

Let the error of the estimate be $\underline{e}(j) = \underline{x}(j) - \hat{\underline{x}}(j)$ where $\underline{x}(j)$ is the true state value at time $t(j)$ and $\hat{\underline{x}}(j)$ is the value of the estimate. Let $L(\underline{e}(j))$ be a cost function. The value of the estimate is chosen so as to minimize the expected value of the cost function:

$\hat{\underline{x}}(j)$ is the value of $\underline{x}(j)$ such that $E[L(\underline{e}(j))]$ is minimum, with

$$E[L(\underline{e}(j))] = \int L(\underline{e}(j))p(\underline{x}(j)/Z^k) d\underline{x}(j) .$$

This expectation is taken over the conditional density function of the states and therein lies the importance of this function.

Table I gives three main cost functions which lead to different choices for the estimate.

<u>Cost Function $L(e(j))$</u>	<u>Name</u>	<u>Estimate $\hat{x}(j)$</u>
1. $e^T(j) \cdot e(j)$	minimum variance or minimum mean square error	Conditional mean
2. $ e(j) $	minimum absolute error	Conditional median
3. $= 0 \quad e(j) < \underline{\delta}$ $= 1 \quad \text{otherwise}$	uniform	Conditional mode or maximum a posteriori estimate

TABLE I. Cost Functions and Their Estimates

2. Recursive Bayesian Formulation

In the discrete formulation of the filtering problem, the conditional density function can be written in a recursive manner with the initial condition probability distribution as the starting function [26].

Let the conditional density of the filter states be $p(\underline{x}(k)/Z^k)$. Using Bayes rule this can be written as:

$$\begin{aligned}
 p(\underline{x}(k)/Z^k) &= p(\underline{x}(k)/\underline{z}(k), Z^{k-1}) = \frac{p(\underline{x}(k), \underline{z}(k), Z^{k-1})}{p(\underline{z}(k), Z^{k-1})} \\
 &= \frac{p(\underline{z}(k)/\underline{x}(k), Z^{k-1}) \cdot p(\underline{x}(k), Z^{k-1})}{p(\underline{z}(k)/Z^{k-1}) \cdot p(Z^{k-1})} \\
 p(\underline{x}(k)/Z^k) &= \frac{p(\underline{z}(k)/\underline{x}(k))}{p(\underline{z}(k)/Z^{k-1})} \cdot p(\underline{x}(k)/Z^{k-1}) \quad (2.7)
 \end{aligned}$$

Thus the current value of the conditional density function is found from the predicted density $p(\underline{x}(k) / Z^{k-1})$ and a weighting term which depends on the current observation. Since the denominator term does not depend on the states, $\underline{x}(k)$, it can be looked on as a normalizing constant[10].

The predicted conditional density function can be found from the density function one step back in the following manner:

$$\begin{aligned}
 p(\underline{x}(k) / Z^{k-1}) &= \int_{\underline{x}(k-1)} p(\underline{x}(k), \underline{x}(k-1) / Z^{k-1}) \\
 p(\underline{x}(k)/Z^{k-1}) &= \int_{\underline{x}(k-1)} \frac{p(\underline{x}(k), \underline{x}(k-1), Z^{k-1})}{p(Z^{k-1})} d\underline{x}(k-1) \\
 &= \int_{\underline{x}(k-1)} \frac{p(\underline{x}(k)/\underline{x}(k-1), Z^{k-1}) p(\underline{x}(k-1), Z^{k-1})}{p(Z^{k-1})} d\underline{x}(k-1) \quad (2.8)
 \end{aligned}$$

Now using the Markov property of the state system,

$$p(\underline{x}(k)/Z^{k-1}) = \int_{\underline{x}(k-1)} p(\underline{x}(k)/\underline{x}(k-1)) p(\underline{x}(k-1)/Z^{k-1}) d\underline{x}(k-1) . \quad (2.9)$$

The notation for the integral indicates that the integration is performed over all permissible values of the states one step back. The predicted conditional density is thus found from the density one step back and involves a spatial

integration over all values of all the states at that time. The final recursive expression for the conditional density is thus

$$p(\underline{x}(k)/Z^k) =$$

$$\frac{p(\underline{z}(k)/\underline{x}(k))}{p(\underline{z}(k)/Z^{k-1})} \cdot \int_{\underline{x}(k-1)} p(\underline{x}(k)/\underline{x}(k-1)) p(\underline{x}(k-1)/Z^{k-1}) d\underline{x}(k-1) \quad (2.10)$$

with the condition $p(\underline{x}(0)/Z^0) = p(\underline{x}(0))$. The individual conditional density functions above are obtained from the probability density functions of the state excitation noise, $\underline{w}(k-1)$, and the observation noise, $\underline{v}(k)$.

Let the density of $\underline{w}(k)$ be $p_{\underline{w}}(\underline{w}(k))$ and the density of $\underline{v}(k)$ be $p_{\underline{v}}(\underline{v}(k))$. Therefore

$$p(\underline{z}(k)/\underline{x}(k)) = p_{\underline{v}}(\underline{z}(k) - \underline{h}(\underline{x}(k), k)) , \quad (2.11)$$

and

$$p(\underline{x}(k)/\underline{x}(k-1)) = p_{\underline{w}}[\underline{g}^{-1}(\underline{x}(k-1), k-1) \cdot [\underline{x}(k) - \underline{f}(\underline{x}(k-1), k-1)]] \quad (2.12)$$

3. Linear Gaussian Case

When the state equations and the observation equations are linear, and the initial density of $\underline{x}(0)$ and the densities of $\underline{w}(k-1)$ and $\underline{v}(k)$ are Gaussian all of the above conditional densities turn out to be Gaussian since

they involve linear operations on Gaussianly distributed random variables. Since these densities are completely determined by two quantities, the vector mean and the covariance matrix, the recursive expression for the conditional density function, Eq. (2.10) simplifies to two coupled expressions: one for the vector mean and the other for the covariance matrix. These two expressions, when rewritten are the well-known Kalman filter equations [36, 22,23].

When the state or observation equations are nonlinear or the noise statistics are non-Gaussian, the above simplification is not valid. Thus the entire density is needed to completely solve the problem.

B. THE CONTINUOUS CASE

The equations for the states and observations can be written in continuous form

$$d\underline{x}(t) = \underline{f}(\underline{x}(t),t)dt + \underline{g}(\underline{x}(t),t)d\underline{w}(t), \quad (2.13)$$

and

$$d\underline{z}(t) = \underline{h}(\underline{x}(t),t)dt + d\underline{v}(t) . \quad (2.14)$$

The above equations are Ito stochastic differential equations and must be manipulated using the rules of Ito calculus [17]. The difficulty with ordinary calculus is in

the definition of integrals involving white noise. The first equation above is to be interpreted as

$$\underline{x}(t) - \underline{x}(t_0) = \int_{t_0}^t \underline{f}(\underline{x}(t), t) dt + \int_{t_0}^t \underline{g}(\underline{x}(t), t) d\underline{w}(t). \quad (2.15)$$

Since the last integral involves integration over random increments ($d\underline{w}(t)$), with random quantities in the integral, it is not even defined by ordinary calculus [21]. The increments appearing in Equations (2.13) and (2.14), $d\underline{w}(t)$ and $d\underline{v}(t)$ are increments of two independent Wiener processes.

When the continuous equations are interpreted in the Ito sense, the process $\underline{x}(t)$ still defines a Markov process and as such, evolution of the density function of $\underline{x}(t)$ is given by the Kolmogorov equations or the Fokker-Planck equation [19,45]. Kushner [25], Bucy [6], and Fisher [18] have shown that the conditional density $P(\underline{x}(t)/Z^t)$ - where Z^t is the set of observations in the interval (t_0, t) - also obeys a Kolmogorov type equation which is modified by the observations. The result is an Ito differential equation for the conditional density which is strictly a formal solution to the problem. By this is meant that no closed form solution can be found because there is no theory to solve the equation [21]. See also Bucy and Joseph [8].

C. SOLUTION METHODS AND APPROXIMATIONS

It is no surprise, and this work is no exception, that most of the practical non-linear filters use the Kalman

filter equations in some linearized system of equations [38]. The linearization is usually carried out about some nominal state trajectory or about a predicted value for the states on a step by step basis. This technique is Extended Kalman filtering. The tacit assumption here is that with Gaussian noise, there is some region in state space around the true state values where the system response is linear and therefore Gaussian. Similarly the observation errors will be linear and Gaussian. The greatest difficulty with this approach is insuring that the linearization is valid, especially at the initial filter estimates where the errors may be quite large. For the passive problem described in the next chapter, the initial conditions are not known and must be obtained from the first few observations with sufficient accuracy to insure filter convergence.

Several authors have looked at higher order terms in the expansions of the state and observations equations [3,5,27,40,43]. This greatly increases the complexity of the calculations and has not been shown to be generally applicable to all problems.

There have been attempts to look at higher moments of the conditional density function itself in an effort to characterize the function at each state point. Fisher [18] has derived equations for the evolution of higher order moments. The difficulty is in determining how many moments are needed to describe the density and in solving the highly

coupled set of nonlinear difference or differential equations which are the result [24].

Srinivasan [41] and Sorensen and Stubberuds [39] expanded the density into an orthogonal series and then looked at the evolution of each of the terms in the expansion. It is difficult to know how many terms to incorporate and in some cases the series may not converge at all [21]. Spline function expansions have been used but have the same inherent difficulties [14]. Swirling, et al. [42] have attempted a logarithmic expansion of the conditional mean.

One of the most successful expansion techniques has been the Gaussian-Sum Expansion of Alspach and Sorenson [1,2]. While the computational burden is quite large, this method is one of the few that will solve multi-modal distributions. The primary reason for its universal application is that each linearization of the state and observation equation only has to be valid about the mean of each of the assumed Gaussian distributions used in the sum. Each term in the sum is then filtered using the linear Kalman filter equations, and the results recombined and normalized to give an approximation to the conditional density.

For limited order systems, Bucy and Senne [7] have done a direct calculation of the conditional density using the Bayes law recursive equations developed in the previous section. Here again the computational burden is enormous and

is only possible with two or three state variables. For these systems, however, this technique does result in the complete solution to the nonlinear filtering problem in the sense that the result is the complete a posteriori conditional density function. The particular value of the state estimate may then be determined using a suitable cost criterion. See also Senne [37] and Bucy, Hecht and Senne [9].

The method of least squares, while being strictly a non-probabilistic approach is widely used in non-linear problems [4,12,20,28,33]. In this method a non-probabilistic cost function is developed which is usually a quadratic form of the state trajectory with some appropriate state model (See for example Sage and Melsa [36]). This cost function is then minimized, using the observed data, with respect to the state trajectory points. As new data is added a recursive algorithm can be used to update the estimates [15,30]. With appropriate weighting matrices used in the cost function, this method can be shown to be exactly equivalent for Gaussian noise statistics to the maximum a posteriori estimate using a probability approach [21,36].

The method of least squares for nonlinear problems is usually solved by linearization about some estimate and then iteration until a solution is found. Thus the initial guess must be sufficiently close to allow for convergence. Also for a large-order system the amount of computation may be extremely large.

The technique involves setting up a suitable cost function as a set of simultaneous non-linear equations which contains the observed data and some set of state parameters as unknowns. In this system of equations (usually over-determined) the "best fit" criteria is applied, generally with a data error norm. This method is used in the doppler-frequency-only localization method [46]. When the system of over-determined equations is linear, this approach results in the pseudo-inverse matrix technique [28].

III. PASSIVE LOCATION AND TRACKING

In the two-dimensional location problem, there are two position and two velocity components which will specify a constant-speed, constant-heading target. Since the doppler shifted frequency is used as an observable, it is also necessary to estimate the rest frequency of the emitter. These five quantities will be used to define two different filter formulations: one in which a target independent X-Y coordinate frame is used and one in which a target dependent system with one axis aligned with the estimated heading is used. Before presenting the two filter types, a general discussion of the Extended Kalman filter is given.

A. EXTENDED KALMAN FILTER

Consider a nonlinear discrete system of state and observation equations given by

$$\underline{x}(k+1) = \underline{f}(\underline{x}(k),k) + \underline{g}(\underline{x}(k),k) \cdot \underline{w}(k) \quad (3.1)$$

and

$$\underline{z}(k) = \underline{h}(\underline{x}(k),k) + \underline{v}(k) . \quad (3.2)$$

In these equations \underline{f} , \underline{g} and \underline{h} are nonlinear functions of the state variables \underline{x} , $\underline{w}(k)$ is plant excitation noise, and $\underline{v}(k)$ is measurement noise. The plant noise and measurement noise are assumed uncorrelated, zero-mean, and white. That is

$$E[\underline{w}(k) \cdot \underline{w}^T(j)] = Q'(k) \delta_{kj}$$

and

$$E[\underline{v}(k) \cdot \underline{v}^T(j)] = R(k) \delta_{kj}$$

In order to apply the linear filter equations, (3.1) and (3.2) are expanded about the best estimate of the state at that time and only the first-order terms are kept.

Equation (3.1) gives

$$\underline{x}(k+1) = \Phi(k)\underline{x}(k) + \Gamma(k) \cdot \underline{w}(k) \quad , \quad (3.3)$$

with

$$\Phi(k) = \left. \frac{\partial \underline{f}}{\partial \underline{x}} \right|_{\underline{x}=\hat{\underline{x}}(k)} \quad (3.3a)$$

Similarly Eq. (3.2) yields

$$\underline{z}(k) = H(k) \underline{x}(k) + \underline{v}(k) \quad (3.4)$$

where

$$H(k) = \left. \frac{\partial \underline{h}}{\partial \underline{x}} \right|_{\underline{x}=\underline{x}'(k)} \quad (3.4a)$$

$\hat{\underline{x}}(k)$ is the estimated state value after the k^{th} measurement and $\underline{x}'(k)$ is the predicted value of the state before the k^{th} measurement. That is,

$$\underline{x}'(k) = \underline{f}(\hat{\underline{x}}(k-1), k-1).$$

A state error vector is defined by

$$\tilde{\underline{x}}(k) = \hat{\underline{x}}(k) - \underline{x}(k) \quad ,$$

and a predicted state error vector is defined by

$$\tilde{\underline{x}}'(k) = \underline{x}'(k) - \underline{x}(k) \quad .$$

The covariance of state error matrix is defined by

$$P(k) = E[\tilde{\underline{x}}(k) \cdot \tilde{\underline{x}}^T(k)] \quad ,$$

and the predicted covariance of state error matrix is given by

$$P'(k) = E[\tilde{\underline{x}}'(k) \cdot \tilde{\underline{x}}'^T(k)]$$

The state excitation ^{cov}matrix is given by

$$Q(k) = E[\Gamma(k) \cdot \underline{w}(k) \cdot \underline{w}^T(k) \cdot \Gamma^T(k)] \quad ,$$

and the measurement noise covariance matrix is

$$R(k) = E[\underline{v}(k) \cdot \underline{v}^T(k)] \quad .$$

The Kalman Filter equations are given by [38]

$$J(t_k/k-1) P'(k+1) = \Phi(k) P(k) \Phi^T(k) + Q(k) \quad (3.5a)$$

$$G(k) = P'(k) H^T(k) [H(k) P'(k) H^T(k) + R(k)]^{-1} \quad (3.5b)$$

$$J(t_k/k) \rightarrow P(k) = [I - G(k) H(k)] P'(k) \quad (3.5c)$$

$$\underline{x}'(k) = \underline{f}(\hat{\underline{x}}(k-1), k) \quad J(t_k/k-1) \quad (3.5d)$$

$$\underline{z}'(k) = \underline{h}(\underline{x}'(k), k) \quad (3.5e)$$

$$\hat{\underline{x}}(k) = \underline{x}'(k) + G(k) [\underline{z}(k) - \underline{z}'(k)] \quad (3.5f)$$

The Q matrix serves not only to allow for maneuvering but also to account for any model inaccuracies. That is, any discrepancies between the true action of the physical system and its characterization by Eq. (3.3). For a filter which reaches steady-state conditions the Q also serves to prevent the gain matrix G(k) from approaching zero by always insuring uncertainty in the predicted covariance of error matrix P'(k).

1. X-Y Filter Equations

Figure 3.1 gives the geometry of the states used for the X-Y filter. For a constant-course, constant-speed target, the plant state equations can be described as two second order systems, one for X and one for Y, and a state for the rest frequency, F_0 . These are

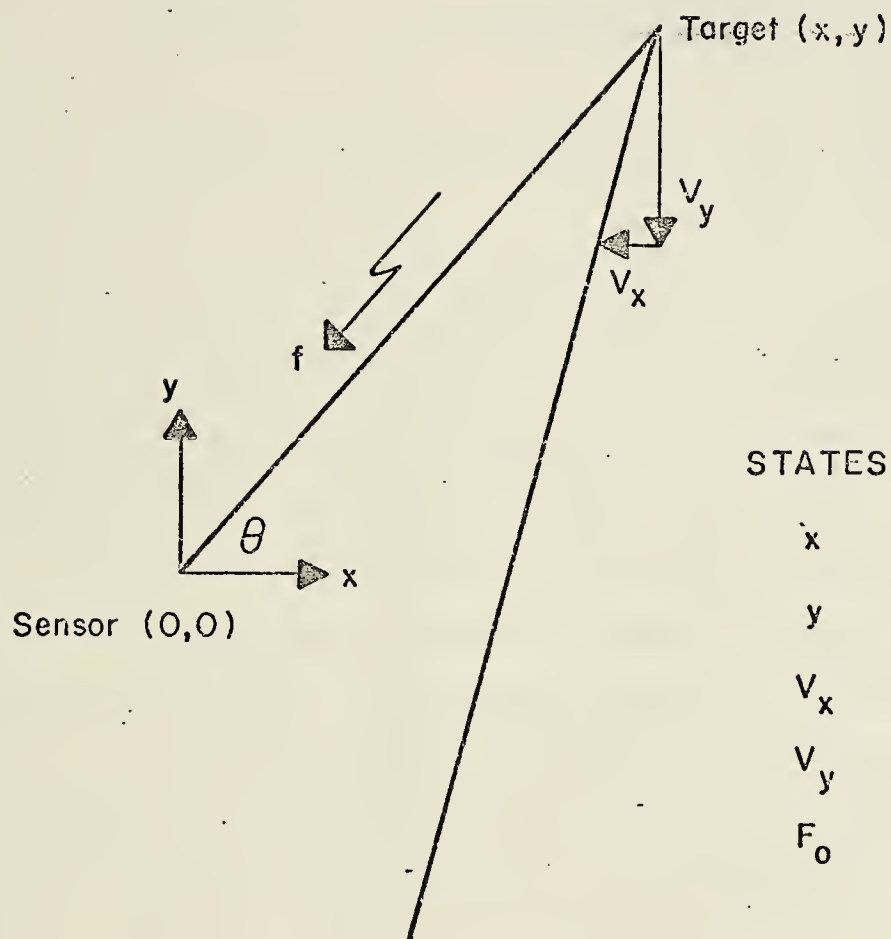


Fig. 3-1 x-y Filter Geometry

$$\underline{x}(k+1) = \begin{bmatrix} x(k+1) \\ v_x(k+1) \\ y(k+1) \\ v_y(k+1) \\ F_o(k+1) \end{bmatrix} = \begin{bmatrix} x(k) + T v_x(k) + f_1(\gamma_{\dot{\theta}_s}, \gamma_{\dot{v}_s}, k) \\ v_x(k) + f_2(\gamma_{\dot{\theta}_s}, \gamma_{\dot{v}_s}, k) \\ y(k) + T v_y(k) + f_3(\gamma_{\dot{\theta}_s}, \gamma_{\dot{v}_s}, k) \\ v_y(k) + f_4(\gamma_{\dot{\theta}_s}, \gamma_{\dot{v}_s}, k) \\ F_o(k) + f_5(\gamma_{\dot{F}_o}) \end{bmatrix} \quad (3.6)$$

where $x(k)$, and $y(k)$ are the position coordinates at time $t(k)$, $v_x(k)$ and $v_y(k)$ are the X and Y components of velocity, T is the time between observation, i.e., $t(k+1) - t(k)$, and $F_o(k)$ is the rest frequency of the emitter.

The excitation terms f_1 through f_5 are included to account for the random changes in speed and heading and F_o which can occur for a maneuvering target. The quantities $\gamma_{\dot{\theta}_s}$, $\gamma_{\dot{v}_s}$ and $\gamma_{\dot{F}_o}$ are the random changes of the target. They are assumed to be independent, zero mean, piecewise-constant rates of change. They have variances defined by

$$\sigma_{\dot{v}_s}^2 = E[\gamma_{\dot{v}_s}^2]$$

$$\sigma_{\dot{\theta}_s}^2 = E[\gamma_{\dot{\theta}_s}^2]$$

$$\sigma_{\dot{F}_o}^2 = E[\gamma_{\dot{F}_o}^2]$$

The values for the standard deviations were taken from typical maneuvering parameters for the target;

$$\begin{aligned}\sigma_{\dot{\theta}_s} &= 100^\circ/1000 \text{ sec} && .01^\circ/\text{sec} \\ \sigma_{\dot{v}_s} &= 10 \text{ kts}/1000 \text{ sec} && .01 \text{ kts}/\text{sec} \\ \sigma_{\dot{F}_0} &= .5 \text{ Hz}/1000 \text{ sec.} && .005 \text{ Hz}/\text{sec}\end{aligned}$$

Handwritten notes: "wrong" for the first two, "Hz" for the third.

The effect of this excitation is to increase the predicted value of the covariance of the state error matrix.

Writing the equations in linear state form results in

$$\underline{x}(k+1) = \Phi \underline{x}(k) + \Gamma \underline{w}(k) \quad + \text{coupling Matrix (forcing functions)} \quad (3.7)$$

where

$$\Phi = \begin{bmatrix} 1 & T & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\Gamma = \begin{bmatrix} T & \frac{T^2}{2} & 0 & 0 & 0 \\ 0 & T & 0 & 0 & 0 \\ 0 & 0 & T & \frac{T^2}{2} & 0 \\ 0 & 0 & 0 & T & 0 \\ 0 & 0 & 0 & 0 & T \end{bmatrix}$$

Handwritten notes: "only" above the matrix, and a circled "2" next to the second column.

The vector $\underline{w}(k)$ represents the effect on the states of the random excitations and may be calculated from the equations relating target X and Y velocity to the target heading and velocity. The X velocity is

$$\dot{x} = v_x = v_s \cdot \cos \theta_s \quad (3.8)$$

which when differentiated gives

$$\ddot{x} = \gamma_{v_s} \cdot \cos \theta_s - v_s \cdot \sin \theta_s \cdot \gamma_{\theta_s} = \frac{v_x}{v_s} \gamma_{v_s} - v_y \gamma_{\theta_s} \quad (3.9)$$

Similarly

$$\dot{y} = v_y = v_s \cdot \sin \theta_s \quad (3.10)$$

and

$$\ddot{y} = \frac{v_y}{v_s} \gamma_{v_s} + v_x \gamma_{\theta_s} \quad (3.11)$$

The frequency term is just

$$\dot{F}_O = \gamma_{F_O} \quad (3.12)$$

Thus

$$\underline{w}(k)^T = \left[0 \quad \gamma_{v_s} \cdot \frac{v_x}{v_s} - \gamma_{\theta_s} \cdot v_y \quad 0 \quad \gamma_{v_s} \cdot \frac{v_y}{v_s} + \gamma_{\theta_s} \cdot v_x \quad \gamma_{F_O} \right] \quad (3.13)$$

where from the assumption on the γ 's

$$E[\underline{w}(k)] = 0 \quad (3.14)$$

The excitation covariance matrix is thus found from

$$Q = E[\Gamma \underline{w}(k) \underline{w}(k)^T \Gamma^T] \quad (3.15)$$

Let

$$\sigma_{\dot{x}}^{..2} = \left(\frac{v_x}{v_s}\right)^2 \sigma_{\dot{v}_s}^{..2} + v_y^2 \sigma_{\dot{\theta}_s}^{..2} \quad (3.16a)$$

$$\sigma_{\dot{y}}^{..2} = \left(\frac{v_y}{v_s}\right)^2 \sigma_{\dot{v}_s}^{..2} + v_x^2 \sigma_{\dot{\theta}_s}^{..2} \quad (3.16b)$$

$$\text{and } \sigma_{xy}^{...} = v_x v_y \left[\frac{\sigma_{v_s}^{..2}}{v_s^2} - \sigma_{\dot{\theta}_s}^{..2} \right] \quad (3.16c)$$

where the states are evaluated at the current state estimates, $\hat{\underline{x}}(k)$. Substituting these expressions in the Q matrix results in

$$Q = \begin{bmatrix} \left(\frac{T^2}{2}\right)^2 \sigma_{\dot{x}}^{..2} & \frac{T^3}{2} \sigma_{\dot{x}}^{..2} & \left(\frac{T^2}{2}\right) \sigma_{xy}^{...} & \frac{T^3}{2} \sigma_{xy}^{...} & 0 \\ & T^2 \sigma_{\dot{x}}^{..2} & \frac{T^3}{2} \sigma_{xy}^{...} & T^2 \sigma_{xy}^{...} & 0 \\ & & \left(\frac{T^2}{2}\right)^2 \sigma_{\dot{y}}^{..2} & \frac{T^3}{2} \sigma_{\dot{y}}^{..2} & 0 \\ & & & T^2 \sigma_{\dot{y}}^{..2} & 0 \\ \text{symmetric} & & & & T^2 \sigma_{\dot{\theta}_s}^{..2} \end{bmatrix} \quad (3.17)$$

The excitation matrix serves not only to take into account the possibility of maneuvering, but of model inaccuracies as well. Q is also used to prevent the gains

of the filter from approaching zero as more and more data is processed, by insuring some uncertainty in the predicted state values. (See Sage and Melsa [36], page 415.)

The observation equations are nonlinear in the states and are given by

$$\underline{z}(k) = \begin{bmatrix} \theta(k) \\ f(k) \end{bmatrix} = \begin{bmatrix} \tan^{-1} \left[\frac{y(k)}{x(k)} \right] + v_{\theta}(k) \\ \frac{F_o(k) v_p}{v_p + \frac{v_s}{v_p} \cos(\theta_s(k) - \theta(k))} + v_f(k) \end{bmatrix} \quad (3.18a)$$

$$(3.18b)$$

The doppler equation can be expressed in terms of the state variables to give

$$f(k) = \frac{F_o(k) v_p}{v_p + \frac{x(k) v_x(k) + y(k) v_y(k)}{(x(k)^2 + y(k)^2)^{1/2}}} + v_f(k) \quad (3.19)$$

The measurement noises, $v_{\theta}(k)$ and $v_f(k)$, are assumed to be zero-mean and independent with a covariance matrix

$$R(k) = \begin{bmatrix} \sigma_{\theta}^2 & 0 \\ 0 & \sigma_f^2 \end{bmatrix} \quad (3.20)$$

The magnitude of the angle noise is a function of the processor used and the signal-to-noise ratio at the

sensor. A typical accuracy of \pm five degrees is common for strong signals and this value was used as the standard deviation for most of the simulation runs.

The resolution of the frequency measurements is equal to the inverse of the record length of the time signal. For a 25-second record this would be .04 Hz and this value was used throughout the simulation as the frequency noise standard deviation. This value would represent a one-bin uncertainty in the coefficients of a fast Fourier transform processor which typically could compute the transform in less than 100 ms.

Equation (3.4a) can be used to give the linearized observation matrix. The result is

$$H(k) = \begin{bmatrix} \frac{\partial \theta(k)}{\partial x} & \frac{\partial \theta(k)}{\partial v_x} & \frac{\partial \theta(k)}{\partial y} & \frac{\partial \theta(k)}{\partial v_y} & \frac{\partial \theta(k)}{\partial F_o} \\ \frac{\partial f(k)}{\partial x} & \frac{\partial f(k)}{\partial v_x} & \frac{\partial f(k)}{\partial y} & \frac{\partial f(k)}{\partial v_y} & \frac{\partial f(k)}{\partial F_o} \end{bmatrix}$$

When the derivatives are taken and evaluated at the predicted state values $\underline{x}'(k)$ the result is

$$\begin{array}{|c|c|}
\hline
-\frac{y'(k)}{x'(k)^2+y'(k)^2} & 0 \\
\hline
-\frac{f'(k)^2}{F'_0(k)v_p} \cdot \frac{y'(k)[y'(k)v'_x(k)-x'(k)v'_y(k)]}{[x'(k)^2+y'(k)^2]^{3/2}} & -\frac{f'(k)^2}{F'_0(k)v_p} \cdot \frac{x'(k)}{[x'(k)^2+y'(k)^2]^{1/2}} \\
\hline
\end{array}$$

$$\begin{array}{|c|c|}
\hline
\frac{x'(k)}{x'(k)^2+y'(k)^2} & 0 \\
\hline
-\frac{f'(k)}{F'_0(k)v_p} \cdot \frac{x'(k)[x'(k)v'_y(k)-y'(k)v'_x(k)]}{[x'(k)^2+y'(k)^2]^{3/2}} & -\frac{f'(k)^2}{F'_0(k)v_p} \cdot \frac{y'(k)}{[x'(k)^2+y'(k)^2]^{1/2}} \\
\hline
\end{array}$$

$$\begin{array}{|c|c|}
\hline
0 & \\
\hline
\frac{f'(k)}{F'_0(k)} & \\
\hline
\end{array}$$

$$(3.22)$$

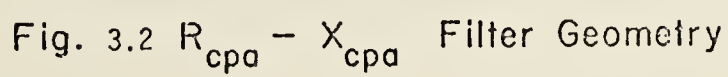
In the X-Y filter, both position coordinates will change as functions of time. Furthermore, a change in either heading or speed will result in a change in both

velocity components. Thus, the calculation of the state excitation matrix, Q , involves all state related terms except for those involving F_o . Since a constant-speed, constant-heading target is assumed, a different set of states can be found which will decouple the heading and speed excitation and involve the estimation of only one linearly, time-varying quantity.

Let R_{cpa} be the range to the target at the closest point of approach (cpa), X_{cpa} be the distance to cpa (negative before — positive after), v_s be target speed, θ_s be target heading and F_o be target rest frequency. The geometry is shown in Fig. 3.2. Expressed in this coordinate system the state equations are

$$\begin{aligned}
 R_{cpa}(k+1) &= R_{cpa}(k) + g_1(\dot{\gamma}_{\theta_s}, \dot{\gamma}_{v_s}, k) \\
 X_{cpa}(k+1) &= X_{cpa}(k) + v_s \cdot T + g_2(\dot{\gamma}_{\theta_s}, \dot{\gamma}_{v_s}, k) \\
 v_s(k+1) &= v_s(k) + g_3(\dot{\gamma}_{v_s}) \\
 \theta_s(k+1) &= \theta_s(k) + g_4(\dot{\gamma}_{\theta_s}) \\
 F_o(k+1) &= F_o(k) + g_5(\dot{\gamma}_{F_o})
 \end{aligned} \tag{3.23}$$

with g_1 through g_5 the random forcing terms.



The state transition matrix is therefore

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ & 1 & T & 0 & 0 \\ & & 1 & 0 & 0 \\ & 0 & & 1 & 0 \\ & & & & 1 \end{bmatrix}$$

The angle observation equation is

$$\begin{aligned} \theta(k) &= \theta_s(k) - \tan^{-1} \left[\frac{A \cdot R_{cpa}(k)}{X_{cpa}(k)} \right] + v_\theta(k) && \text{if } X_{cpa} > 0 \\ &= \theta_s(k) - \tan^{-1} \left[\frac{A \cdot R_{cpa}(k)}{X_{cpa}(k)} \right] - 180 + v_\theta(k) && X_{cpa} < 0 \end{aligned}$$

(3.24)

A is an angular rotation term needed to give the correct sign for a given geometry. A is +1 for counter clockwise rotation about the sensor and -1 for clockwise rotation.

The frequency observation equation is

$$f(k) = \frac{F_o(k) \cdot v_p}{v_p + \frac{v_s(k) \cdot X_{cpa}(k)}{[R_{cpa}(k)^2 + X_{cpa}(k)^2]^{1/2}}} + v_f(k) \quad (3.25)$$

The excitation matrix, Q, for the states is found in a similar manner to that for the X-Y coordinate system. A heading change effects both R_{cpa} and X_{cpa} while a velocity change only effects X_{cpa} . The random excitation will again be modeled as zero mean, piecewise-constant random heading-angle velocity, random change in speed, and random change in rest frequency. With the expressions for the random forcing terms included, the state equations become

$$R_{cpa}(k+1) = R_{cpa}(k) + A \cdot X_{cpa}(k) \cdot \gamma_{\dot{\theta}_s} \cdot T$$

$$X_{cpa}(k+1) = X_{cpa}(k) + v_s(k) \cdot T + \frac{1}{2} \gamma_{\dot{v}_s} T^2 - A \cdot R_{cpa}(k) \gamma_{\dot{\theta}_s} \cdot T$$

$$v_s(k+1) = v_s(k) + \gamma_{\dot{v}_s} \cdot T \quad (3.26)$$

$$\theta_s(k+1) = \theta_s(k) + \gamma_{\dot{\theta}_s} \cdot T$$

$$F_o(k+1) = F_o(k) + \gamma_{\dot{F}_o} \cdot T \quad .$$

The Q matrix can thus be found and is

$$Q(k) = \begin{bmatrix} X_{cpa}^2 \cdot T^2 \cdot \sigma_{\dot{\theta}_s}^2 & R_{cpa} X_{cpa} T^2 \sigma_{\dot{\theta}_s}^2 & 0 & -A \cdot X_{cpa} T^2 \sigma_{\dot{\theta}_s}^2 & 0 \\ & (\frac{T^2}{2})^2 \sigma_{\dot{v}_s}^2 + R_{cpa}^2 T^2 \sigma_{\dot{\theta}_s}^2 & \frac{T^3}{2} \sigma_{\dot{v}_s}^2 & -A \cdot R_{cpa} T^2 \sigma_{\dot{\theta}_s}^2 & 0 \\ & & T^2 \sigma_{\dot{v}_s}^2 & 0 & 0 \\ & & & T^2 \sigma_{\dot{\theta}_s}^2 & 0 \\ & \text{Symmetric} & & & T^2 \sigma_{\dot{F}_o}^2 \end{bmatrix} \quad (3.27)$$

Here only the first two rows involve state related terms.

The nonlinear observation expressions are obtained the same way as in the X-Y filter. The resulting matrix is

$$H(k) = \begin{bmatrix} \frac{\partial \theta(k)}{\partial R_{cpa}} & \frac{\partial \theta(k)}{\partial X_{cpa}} & \frac{\partial \theta(k)}{\partial v_s} & \frac{\partial \theta(k)}{\partial \theta_s} & \frac{\partial \theta(k)}{\partial F_o} \\ \frac{\partial f(k)}{\partial R_{cpa}} & \frac{\partial f(k)}{\partial X_{cpa}} & \frac{\partial f(k)}{\partial v_s} & \frac{\partial f(k)}{\partial \theta_s} & \frac{\partial f(k)}{\partial F_o} \end{bmatrix} \quad (3.28)$$

$$H(k) = \begin{bmatrix} -\frac{A \cdot X'_{cpa}(k)}{R'(k)^2} & -\frac{A \cdot R'_{cpa}(k)}{R'(k)^2} \\ \frac{f'(k)^2 \cdot v'_s(k)}{F'_o(k) v_p} \cdot \frac{R'_{cpa}(k) \cdot X'_{cpa}(k)}{R'(k)^3} & -\frac{f'(k)^2 \cdot v'_s(k)}{F'_o(k) v_p} \cdot \frac{R'^2_{cpa}}{R'(k)^3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -\frac{f'(k)^2}{F'_o(k) v_p} \cdot \frac{X'_{cpa}(k)}{R'(k)} & 0 & \frac{f'(k)}{F'_o(k)} \end{bmatrix}, \quad (3.29)$$

where $f'(k)$ is the predicted frequency measurement and

$$R'(k) = [R'^2_{cpa}(k) + X'^2_{cpa}(k)]^{1/2}, \quad (3.30)$$

is the predicted range.

The Φ , Q , H , and R matrices are again substituted in the Kalman filter equations at each time point.

B. SINGLE SENSOR INITIALIZATION

For a deterministic problem, since there are five unknowns, a set of five independent measurements or equations are necessary. With a sensor which measures signal arrival angle and signal frequency, this would require measurements at three different times to give six equations. Because of the transcendental functions in the doppler expression,

these equations cannot be solved in closed form to give the unknowns, and some type of numerical technique for the solution of nonlinear equations must be used. The addition of the measurement noise complicates the situation further, especially for the angle measurement since the angle noise may be large.

To reduce the effect of noise, the difference in the angle measurements at different time points should be as large as possible. Appendix C shows the probability of having a given error in the angle difference measurements as a function of the measured angle difference, with the measured difference in standard deviations of the noise. Figure C1 shows the larger the difference relative to the expected value of angle noise, the more accurate will the measurement solution be. This may be difficult to obtain for three time points in a practical situation, since the target may not be held for a long enough time to show a large angle change due to fading signals or convergence zone propagation.

There is an alternate method whereby an independent measurement can be obtained by looking at the rate of change of the doppler shifted frequency with time. The rate of change can be approximated with finite difference expressions and, because of range dependence in the derivative, the range can be calculated with measurements at only two different times. Furthermore the heading of the target can be found if the speed of the target is assumed known. The condition

is usually known to sufficient accuracy to give a good heading estimate.

The Doppler equation is

$$f = \frac{F_o}{1 + \frac{v_s}{v_p} \cos(\theta_s - \theta)} \quad (3.31)$$

where f is the doppler shifted frequency, F_o is the rest frequency, v_s is the target speed, v_p is the velocity of propagation of the signal, θ_s is the target heading, and θ is the signal angle of arrival.

Taking the derivative gives

$$\frac{df}{dt} = - \frac{F_o}{[1 + \frac{v_s}{v_p} \cos(\theta_s - \theta)]^2} \cdot \left(- \frac{v_s}{v_p} \sin(\theta_s - \theta) \cdot \left(- \frac{d\theta}{dt} \right) \right) , \quad (3.32)$$

which reduces to

$$\frac{df}{dt} = - \frac{f^2}{F_o v_p} \cdot v_s \sin(\theta_s - \theta) \cdot \frac{d\theta}{dt} \quad (3.33)$$

The transverse velocity about the sensor is

$$R_o \frac{d\theta}{dt} = v_s \sin(\theta_s - \theta) , \quad (3.34)$$

with R_o the target range, which when substituted into the previous equation gives

$$\frac{df}{dt} = - \frac{f^2}{F_o v_p} \cdot R_o \left(\frac{d\theta}{dt}\right)^2 \quad (3.35)$$

Solving for the range R_o yields

$$R_o = - \frac{F_o v_p}{f^2} \cdot \frac{df}{dt} \cdot \frac{1}{(d\theta/dt)^2} \quad (3.36)$$

Substituting finite differences for the derivatives gives

$$R_o \approx - \frac{F_o \cdot v_p \cdot \Delta f \cdot \Delta t}{f^2 (\Delta \theta)^2} \quad (3.37)$$

Since $F_o/f \approx 1$ this gives

$$R_o \approx - \frac{v_p \cdot \Delta f \cdot \Delta t}{f \cdot (\Delta \theta)^2} \quad (3.38)$$

Equation (3.33) can be solved for the heading by writing

$$v_s \sin(\theta_s - \theta) = - \frac{F_o \cdot v_p \cdot (df/dt)}{f^2 (d\theta/dt)} \quad (3.39)$$

and by substituting finite differences for the derivatives,
and assuming some value for v_s , the result is

$$\sin(\theta_s - \theta) \approx - \frac{v_p \cdot \Delta f}{f \cdot \Delta \theta \cdot v_s} \quad (3.40)$$

There are two solutions to this equation since $\sin(\theta) = \sin(180-\theta)$. These two solutions correspond to the up and down doppler case, that is

$$\theta_{s \text{ UP}} = 180 + \theta - \sin^{-1}\left[-\frac{v_p \Delta f}{f \cdot \Delta \theta \cdot v_s}\right] \quad (3.41a)$$

and

$$\theta_{s \text{ DOWN}} = \theta + \sin^{-1}\left[-\frac{v_p \cdot \Delta f}{f \cdot \Delta \theta \cdot v_s}\right] \quad (3.41b)$$

The exact difference equation solutions are presented in Appendix A. The derivatives above have been approximated at the angle midway between the two data points. Figure 3.3 shows the geometry for the calculation. Note that the range can be found independently of velocity. These four quantities, the target range and target heading, the target bearing and assumed velocity are sufficient to initialize the extended Kalman filters because all of the initial states can be calculated from them.

The first quantity to be calculated is the initial target heading θ_{sI} , from the equation (3.41a) or (3.41b). The value of the angle difference, $\Delta\theta$, comes from the two angle observations which satisfy the criterion of being greater than three standard deviations of the measurement noise apart. With this condition and assuming normally distributed measurement errors, Appendix C shows that the probability that the fractional error in the angle measurement, $\theta_e/\Delta\theta$, where θ_e is the measurement error, and $\Delta\theta$ is the measured angle

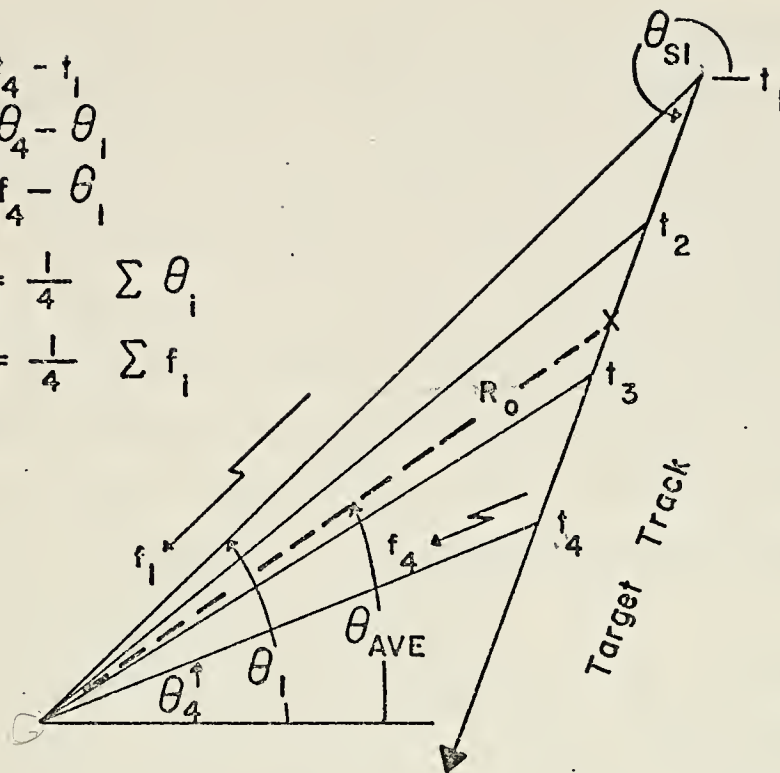
$$\Delta t = t_4 - t_1$$

$$\Delta \theta = \theta_4 - \theta_1$$

$$\Delta f = f_4 - \theta_1$$

$$\theta_{AVE} = \frac{1}{4} \sum \theta_i$$

$$F_{AVE} = \frac{1}{4} \sum f_i$$



$$|\Delta \theta| \geq 3 \sigma_{\theta}$$

$$\theta_{Sl} = \theta_{AVE} - 180 - \sin^{-1} \left[- \frac{v_p \Delta f}{\Delta \theta v_{Sl} f_4} \right]$$

$$R_0 = - \frac{v_p \Delta f \Delta t}{(\Delta \theta)^2 f_4}$$

$$F_{OI} = F_{AVE} \left[1 + \frac{v_{Sl}}{v_p} \cos (\theta_{Sl} - \theta_{AVE}) \right]$$

Fig. 3.3 Calculation of Initial Range, Heading, and Rest Frequency

difference, is less than .5 is equal to .7. This criterion was somewhat arbitrary but was found to give satisfactory results in most instances. It represents a compromise between good initial values and long waiting times to start the filter. Note that instead of using only the endpoint measurements to calculate the midpoint angle, any other measurements of the angle taken in that interval may be used. The appropriate weighted average is used to obtain the result. For periodic measurements this is just the simple average of the measurements. The midpoint angle calculated in this manner is referred to as θ_{AVE} .

The value of the frequency difference, Δf , is obtained by subtracting the first and last frequency measurements obtained when the angle condition is satisfied. The quantity v_p comes from the known propagation velocity, f is taken as one of the measured frequencies, and v_{sI} is the initial guess at the target speed. When the quantity

$$\sin(\theta_{sI} - \theta_{AVE}) = - \frac{v_p \cdot \Delta f}{\Delta \theta \cdot f \cdot v_{sI}} \quad (3.42)$$

is calculated and its absolute value is greater than one, a correction on the velocity guess may be made to bring $|\sin(\theta_{sI} - \theta_{AVE})|$ less than one. Geometrically this means that for the quantities measured, the speed of the target was not large enough to traverse the angle measured.

When this is done θ_{sI} may be computed using equation (3.41a) or (3.41b). The approximate knowledge of the rest frequency and the frequency measurements are usually sufficient to determine whether the target is up or down doppler. The range to the target can then be calculated by using equation (3.38).

The initial rest frequency can be calculated by averaging the doppler equations, again over the time that $\Delta\theta$ was being measured. The doppler equation is

$$f_i = \frac{F_{oI}}{1 + \frac{v_{sI}}{v_p} \cos(\theta_{sI} - \theta_i)} \quad , \quad (3.43)$$

where f_i and θ_i refer to the i^{th} measurement in the start-up interval. F_{oI} is the initial rest frequency to be calculated. Solving for the initial rest frequency F_{oI} gives

$$F_{oI} = f_i \left[1 + \frac{v_{sI}}{v_p} \cos(\theta_{sI} - \theta_i) \right] \quad , \quad (3.44)$$

and averaging over the initial KJ measurements gives

$$F_{oI} = \frac{1}{KJ} \sum_{i=1}^{KJ} f_i \left[1 + \frac{v_{sI}}{v_p} \cos(\theta_{sI} - \theta_i) \right] \quad , \quad (3.45)$$

which is, for small angle differences between angle measurements, approximately

$$F_{oI} \approx F_{AVE} \left[1 + \frac{v_{sI}}{v_p} \cos(\theta_{sI} - \theta_{AVE}) \right] \quad . \quad (3.46)$$

Once the initial range, heading and rest frequency are calculated, the initial conditions for each filter can be found. For the X-Y filter the equations for the initial filter states are

$$\begin{aligned}
 x_{10} &= x_o = R_o \cos \theta_{AVE} \\
 x_{20} &= v_{xo} = v_{sI} \cos \theta_{sI} \\
 x_{30} &= y_o = R_o \sin \theta_{AVE} \\
 x_{40} &= v_{yo} = v_{sI} \sin \theta_{sI} \\
 x_{50} &= F_{oI}
 \end{aligned} \tag{3.47}$$

For the R_{cpa} - X_{cpa} filter the initial states are

$$\begin{aligned}
 x_{10} &= R_{cpaI} = A \cdot R_o \sin(\theta_{sI} - \theta_{AVE}) \\
 x_{20} &= X_{cpaI} = R_o \cos(\theta_{sI} - \theta_{AVE}) \\
 x_{30} &= v_{sI} \\
 x_{40} &= \theta_{sI} \\
 x_{50} &= F_{oI}
 \end{aligned} \tag{3.48}$$

where A, as before is the rotation parameter.

The initial states which are calculated by this technique are the estimated state values at a time approximately midway between the two end measurement times and at an angle

equal to θ_{AVE} . The state equations are now used to bring the states back to the first measurement f_1 and θ_1 and filtering begins with this value. In this manner all of the angle and frequency measurements that were not used in the initialization phase can now be processed. These would correspond to the time points $t+T$ and $t+2T$ in Figure 3.3.

One result of this initialization technique is to correlate the measurements in the interval with those states which are a direct function of θ_{AVE} or F_{AVE} . This violates the assumption usually made for Kalman type filters which is that all measurement noises are uncorrelated with the initial filter states.

This effect decreases as the number of points, KJ , in the initialization interval increases. This is due to the averaging that was used to calculate the midpoint angle and frequency. Consider the covariance between an angle, θ_i , in the interval, and the average over that interval, θ_{AVE} , which is used for position determination. The covariance is

$$\text{cov}[\theta_i, \theta_{AVE}] = E[(\theta_i - \theta_{i0}) \cdot (\theta_{AVE} - \theta_{AVE0})], \quad (3.49)$$

where θ_{i0} and θ_{AVE0} are the mean values. Writing out the terms gives

$$\begin{aligned} \text{cov}(\theta_i, \theta_{AVE}) = E[(\theta_i - \theta_{i0}) \\ \cdot (\frac{\theta_1 + \theta_2 + \dots + \theta_1 + \theta_{KJ}}{KJ} - \frac{\theta_{10} + \theta_{20} + \dots + \theta_{10} + \dots + \theta_{KJ0}}{KJ})] \end{aligned} \quad (3.50)$$

which can be rearranged to get

$$\begin{aligned} \text{cov}(\theta_i, \theta_{\text{AVE}}) &= E[(\theta_i - \theta_{i0}) \\ &\quad \cdot (\frac{\theta_1 - \theta_{10}}{KJ} + \frac{\theta_2 - \theta_{20}}{KJ} + \dots + \frac{\theta_i - \theta_{i0}}{KJ} + \dots + \frac{\theta_{KJ} - \theta_{KJ0}}{KJ})] \end{aligned} \quad (3.51)$$

Thus, the covariance is

$$\text{cov}(\theta_i, \theta_{\text{AVE}}) = \frac{\sigma_\theta^2}{KJ} \quad , \quad (3.52)$$

with σ_θ^2 the angle noise variance. The same results apply to the frequency measurements and F_{AVE} which is used to calculate the initial F_{OI} .

The other requirement to initialize the filter is the initial value of the covariance matrix. Since all of the initial states for each filter were calculated from the same set of measurements, the initial filter states will be correlated with each other. This correlation shows up as cross terms in the calculated initial covariance matrix and represents the relationship between the initial data and the calculation of the initial states. Put another way, the cross terms help describe the region in state space where the true initial states are most likely to be as specified by the given measurements and the method of initial state calculation.

C. MULTIPLE SENSORS - FREQUENCY ONLY

1. X-Y Filter

To add additional sensors to the filter, the observation equations at the new sensor must be found. Thus, in equations (3.18a) and (3.18b), the predicted x and y distances to the new sensor were used in place of the predicted states x and y. If the new sensor is at x_B, y_B the predicted distances are

$$x'_B = x' - x_B$$

$$y'_B = y' - y_B \quad (3.53)$$

The differentiation is then performed and evaluated at the predicted state values to give additional rows to the H matrix.

With an additional sensor of frequency only a different initialization scheme was used. The rest frequency and target heading, midpoint angle and approximate range, were obtained using the single sensor equation. From the doppler equation for the second sensor an average angle from the sensor to the target can be approximated.

Let θ_i be the unknown angles to the target from the second sensor, measured during the initial measurements on the first sensor. Thus i goes from 1 to KJ. The frequencies f_i are the observed quantities. Thus

$$f_i = \frac{F_{oI} v_p}{v_p + v_{sI} \cos(\theta_{sI} - \theta_i)} \quad (3.54)$$

KJ frequency measurements will be made from the second sensor and it will be assumed that they are made at the same time as the measurements on the first sensor.

Adding the above KJ equations and solving for the cosine cosine terms gives

$$\sum_{i=1}^{KJ} \cos(\theta_{sI} - \theta_i) = \frac{v_p}{v_{sI}} \cdot \sum_{i=1}^{KJ} \left(\frac{F_{oI}}{f_i} - 1 \right) \quad (3.55)$$

Let

$$\cos(\theta_{sI} - \theta_{2AVE}) = \frac{1}{KJ} \sum_{i=1}^{KJ} \cos(\theta_{sI} - \theta_i)$$

and

$$\frac{F_{oI}}{F_{2AVE}} = \frac{1}{KJ} \sum_{i=1}^{KJ} \frac{F_{oI}}{f_i}$$

Therefore,

$$\theta_{2AVE} = \theta_{sI} \pm \cos^{-1} \left[\frac{v_p}{v_{sI}} \left(\frac{F_{oI}}{F_{2AVE}} - 1 \right) \right] \quad (3.56)$$

As illustrated in Fig. 3.4, there are two solutions for the angle from the sensor to the target. Each of these can be used with the average angle over the same time interval measured from the first sensor. The solution to

$$\theta_{2 AVE} - \theta_{sI} = \pm \cos^{-1} \left[\frac{v_p}{v_{sI}} \left(\frac{F_{oI}}{F_{AVE}} - 1 \right) \right]$$

$$R_i = \frac{d \cdot \csc(\phi - \theta_{AVE})}{\csc(\phi - \theta_{AVE}) - \csc(\phi - \theta_{2 AVEi})}$$

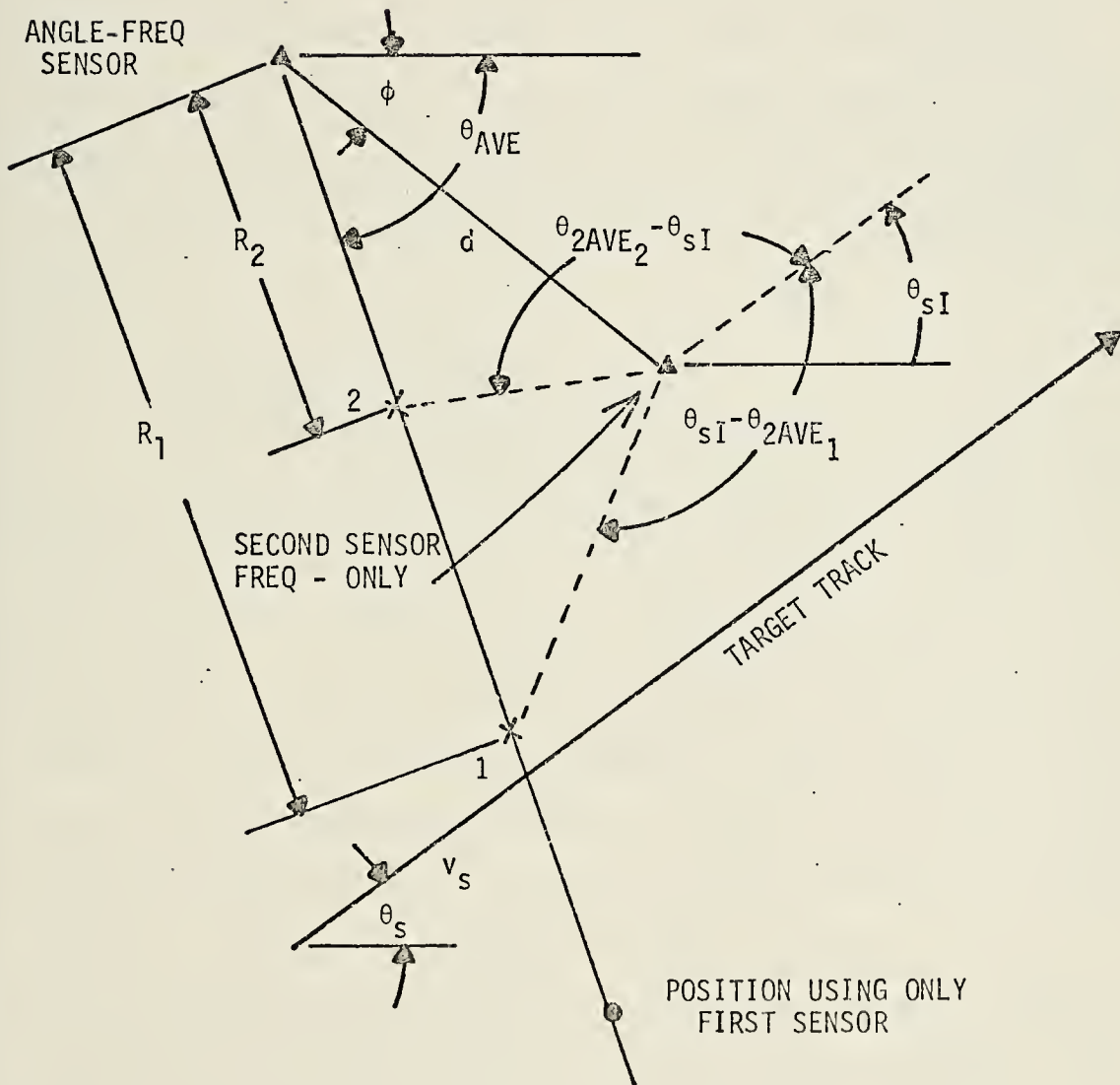


Fig. 3.4 Initial Range with Second Frequency-Only Sensor

this triangulation problem which is closest to the single sensor range is then used as the initial range for the filter.

The reason the second sensor is used at all in the initialization is to insure that the measurements of frequency of the second sensor will be reasonably close to the predicted values that the filter will obtain for its first few samples. If this is not done, the predicted values may be so far off that the linear expansion of the frequency term for the second sensor used for the H matrix is invalid, and the residual will be so large that divergence occurs.

2. $R_{cpa}-X_{cpa}$ Filter

With the $R_{cpa}-X_{cpa}$ filter it is more complicated to add additional sensors because the states were functions of the first sensor only and not of the coordinate system as is the case for the X-Y filter. The derivatives in the observation linearization must be taken with respect to the states of the first buoy. Equations (3.24) and (3.25) for the observed frequency and bearing angle at the sensor are general equations if R_{cpa} and X_{cpa} are referenced to that sensor. As shown in Figure 3.5, R_{cpa_2} and X_{cpa_2} for a second sensor can be written in terms of the R_{cpa} and X_{cpa} of the first sensor, which are states of the filter. In these expressions the angular rotation parameter A , the distance to the second sensor, d , the bearing angle to the



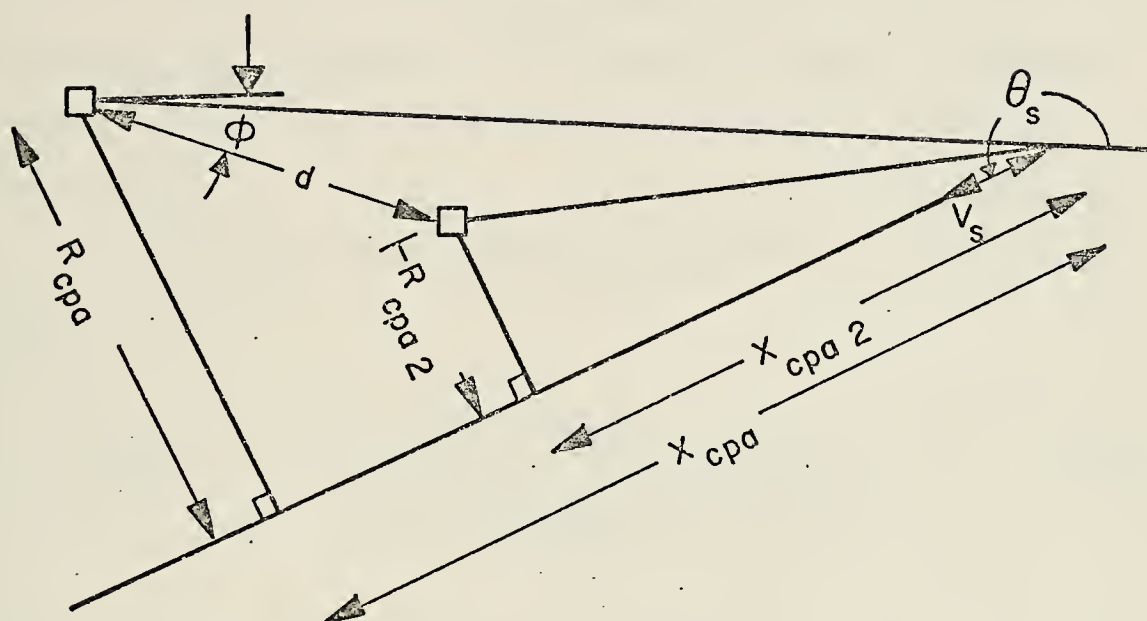


Fig. 3.5 Geometry of 2nd Sensor for $R_{cpa} - X_{cpa}$ Filter.

second sensor ϕ , and the target heading, θ_s , are all significant factors. The result is

$$\begin{aligned} R_{cpa_2} &= R_{cpa} + A \cdot d \cdot \sin(\phi - \theta_s) \\ X_{cpa_2} &= X_{cpa} - d \cos(\phi - \theta_s) \end{aligned} \quad (3.57)$$

Note that this calculation could result in a negative R_{cpa_2} ; however, this indicates that the angular rotation about the second sensor is different from the first sensor.

The resulting observation equation for the second sensor is

$$f_2(k) = \frac{F_o(k) \cdot v_p}{v_p + \frac{v_s(k) \cdot (X_{cpa} - d \cdot \cos(\phi - \theta_s(k)))}{R_2(k)}}, \quad (3.58)$$

where,

$$\begin{aligned} R_2(k)^2 &= (R_{cpa}(k) + A \cdot d \cdot \sin(\phi - \theta_s(k)))^2 \\ &+ (X_{cpa}(k) - d \cdot \cos(\phi - \theta_s(k)))^2, \end{aligned} \quad (3.59)$$

Also,

$$\theta_2(k) = \theta_s(k) - \arctan \left(\frac{A \cdot (R_{cpa}(k) + A \cdot d \cdot \sin(\phi - \theta_s(k)))}{X_{cpa} - d \cdot \cos(\phi - \theta_s(k))} \right). \quad (3.60)$$

There is an additional term of -180° in the last equation if X_{cpa} is less than zero, that is, before cpa. These two equations are then differentiated with respect to the filter states to give the new rows to the observation matrix H.

The result, omitting the time argument, is

$$\frac{\partial f_2}{\partial \underline{x}} = [f_{21} \quad f_{22} \quad f_{23} \quad f_{24} \quad f_{25}] \quad (3.61)$$

with

$$f_{21} = \frac{\partial f_2}{\partial R_{cpa}} = \frac{f_2^2 v_s (X_{cpa} - d \cdot \cos(\phi - \theta_s)) (R_{cpa} + A \cdot d \cdot \sin(\phi - \theta_s))}{F_o v_p R_2^3} \quad (3.62)$$

$$f_{22} = \frac{\partial f_2}{\partial X_{cpa}} = - \frac{f_2^2 v_s (R_{cpa} + A \cdot d \cdot \sin(\phi - \theta_s))}{F_o v_p R_2^3} \quad (3.63)$$

$$f_{23} = \frac{\partial f_2}{\partial v_s} = - \frac{f_2^2 (X_{cpa} - d \cdot \cos(\phi - \theta_s))}{F_o \cdot v_p R_2} \quad (3.64)$$

$$f_{24} = \frac{\partial f_2}{\partial \theta_s} = \frac{f_2^2 v_s d}{F_o v_p R_2} [\sin(\phi - \theta_s) - \frac{(X_{cpa} - d \cdot \cos(\phi - \theta_s))(A \cdot R_{cpa} \cos(\phi - \theta_s) + X_{cpa} \sin(\phi - \theta_s))}{R_2^2}] \quad (3.65)$$

$$f_{25} = \frac{\partial f_2}{\partial F_0} = \frac{f_2}{F_0} \quad (3.66)$$

and

$$\frac{\partial \theta_2}{\partial \underline{x}} = [\theta_{21} \quad \theta_{22} \quad \theta_{23} \quad \theta_{24} \quad \theta_{25}] , \quad (3.67)$$

with

$$\theta_{21} = \frac{\partial \theta_2}{\partial R_{cpa}} = - \frac{A(X_{cpa} - d \cdot \cos(\phi - \theta_s))}{R_2^2} \quad (3.68)$$

$$\theta_{22} = \frac{\partial \theta_2}{\partial X_{cpa}} = \frac{A(R_{cpa} + A \cdot d \cdot \sin(\phi - \theta_s))}{R_2^2} \quad (3.69)$$

$$\theta_{23} = \frac{\partial \theta_2}{\partial v_s} = 0 \quad (3.70)$$

$$\theta_{24} = \frac{\partial \theta_2}{\partial \theta_s} = 1 + \frac{d(X_{cpa} \cdot \cos(\phi - \theta_s) - A \cdot R_{cpa} \cdot \sin(\phi - \theta_s) - d)}{R_2^2} \quad (3.71)$$

$$\theta_{25} = \frac{\partial \theta_2}{\partial F_0} = 0 \quad (3.72)$$

IV. DIRECT METHOD OF COVARIANCE CALCULATION

In Appendix B a method of calculating variances for functions of independent random variables was extended to include the calculation of covariances as well. This technique was used in order to calculate the initial covariance matrix for the two types of filters. The resulting values were in some cases an order of magnitude larger than one would expect based on the accuracies of the measurements and the geometry of the problem.

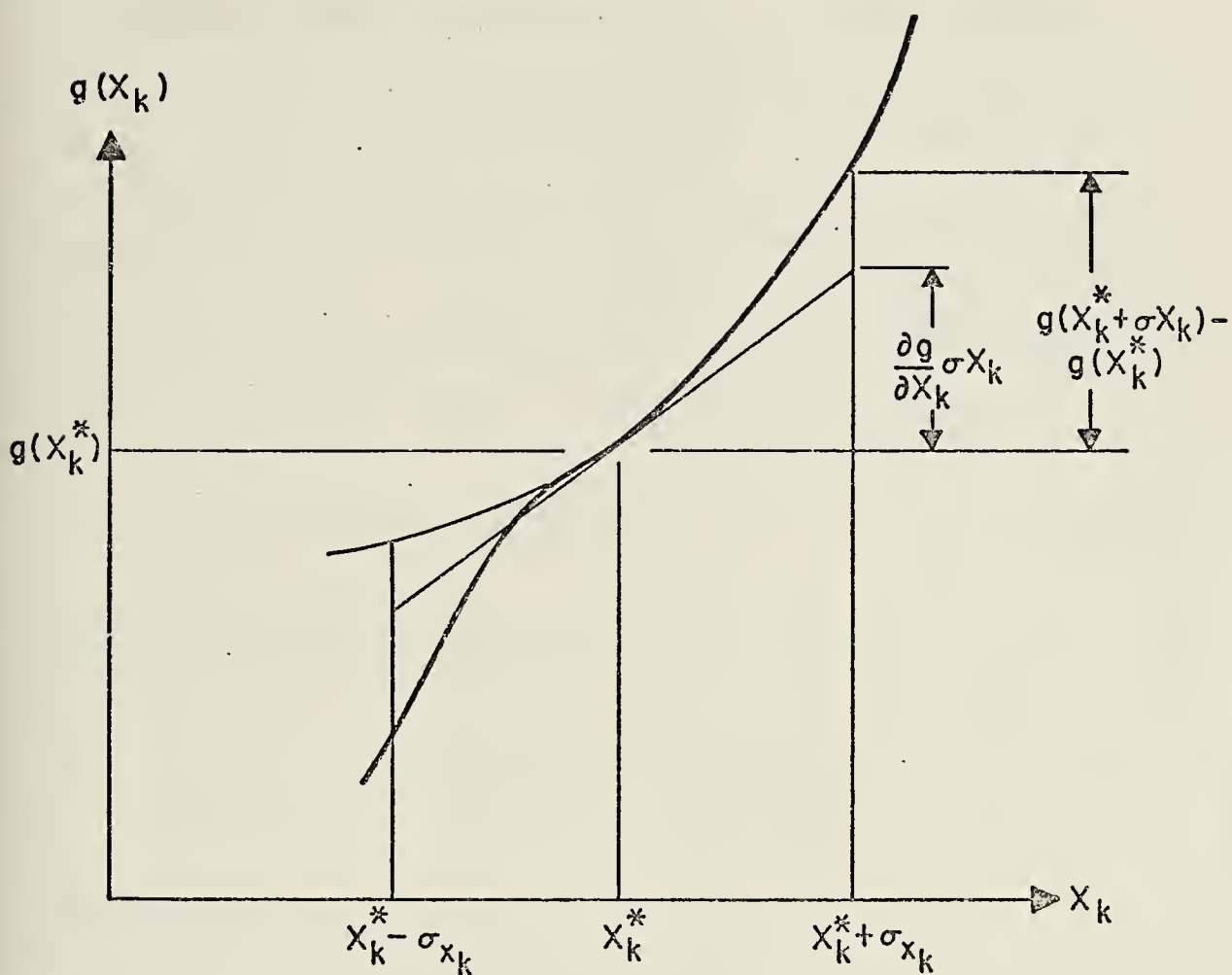
The reason for this is the highly nonlinear nature of the functions involved in the initial condition calculation. The computation of the heading angle, for example, consists of the arcsine function whose derivative approaches infinity as the argument approaches plus or minus one. In reality the range of values for the arcsine function is bounded by plus or minus ninety degrees which represents the target being at cpa. Thus when one uses the derivative formulation the variance or approximate error appears much larger than it really is.

A. INITIAL COVARIANCE MATRIX

An alternate approach was developed which more accurately reflects the error variances of the initial filter conditions. The measurements Δf , $\Delta \theta$, θ_{AVE} , and the initial speed guess v_{SI} , are considered to be Gaussian random variables with known

standard deviations based on the measurement noise and the assumed speed accuracy. The changes that occur in the calculated initial conditions when each of the measurements changes by plus or minus one standard deviation, are measures of the extent over which the initial conditions will vary and thus are an indication of the accuracy of the calculation. These changes can be calculated using the boundary conditions imposed by the geometry of the problem and are not restricted to a linear region or a smooth function as required in the derivative method.

In Fig. 4.1 is shown a one-dimensional example of a nonlinear function $g(x)$. The tangent line drawn at x_k^* is the derivative of the function at x_k^* and its extension by one standard deviation shows the magnitude of the error computed by the derivative method. Also shown is the direct calculation of the changes in $g(x_k)$ when x_k^* is changed by one standard deviation. For this case, the errors one might expect are much larger than the derivative calculation indicates. The difference of this up and down direct calculation, divided by two, is therefore taken as an equivalent "standard deviation" for the calculated value. This is done for each of the three initial measurements and v_{SI} . While no implication is made that this value represents the true standard deviation of the particular initial condition, it does represent, for monotonic functions, the range of values for a one standard deviation change in the measurements. This in turn is a measure of the accuracy of



Replace $\frac{\partial g}{\partial X_k} \sigma_{X_k}$ with $\frac{g(X_k^* + \sigma_{X_k}) - g(X_k^* - \sigma_{X_k})}{Z}$

Fig. 4.1 Taylor Series Expansion Versus Direct Calculation

of the calculation and indeed this is what is ultimately needed for the initial covariance matrix. The off-diagonal terms in this matrix can also be calculated using the method of Appendix B with the derivative times the measurement variances replaced by the calculated "standard deviations" of the initial conditions in equation (B-20). The sign of the covariance terms will be preserved by considering only positive changes in the measurements. An example for the $R_{cpaI}-X_{cpa}$ filter illustrates the procedure. From equation (3.48) the initial range to cpa from the sensor is

$$R_{cpaI} = A \cdot R_o \cdot \sin(\theta_{sI} - \theta_{AVE}) \quad , \quad (4.1)$$

which from equations (3.38) and (3.42) gives

$$R_{cpaI} = A \cdot \left(- \frac{\Delta f \cdot T \cdot v_p}{\Delta \theta^2 \cdot f} \right) \cdot \left(- \frac{\Delta f \cdot v_p}{\Delta \theta \cdot f \cdot v_{sI}} \right) \quad . \quad (4.2)$$

Rearranging terms yields

$$R_{cpaI} = A \cdot \frac{T \cdot v_{sI}}{\Delta \theta} \cdot \left(- \frac{\Delta f \cdot v_p}{\Delta \theta \cdot f \cdot v_{sI}} \right)^2 \quad , \quad (4.3)$$

which can be expressed as

$$R_{cpaI} = A \cdot \frac{T \cdot v_{sI}}{\Delta \theta} \cdot \sin^2(\theta_{sI} - \theta_{AVE}) \quad . \quad (4.4)$$

Written in this form the boundary condition on the value of R_{cpaI} is evident since the sine term is never greater than one.

The equivalent standard deviation in R_{cpaI} due to a one-standard deviation change in $\Delta\theta$, for example, is

$$\sigma_{R_{cpaI}}(\Delta\theta) = \frac{1}{2}[R_{cpaI}(\Delta\theta + \sigma_{\Delta\theta}) - R_{cpaI}(\Delta\theta - \sigma_{\Delta\theta})] \quad (4.5)$$

In this case the boundary condition is imposed in the calculation of the sine term in equation (4.4) above and the maximum value for the range is $\frac{T \cdot v_{sI}}{\Delta\theta}$. Thus

$$|\sin(\theta_{sI} - \theta_{AVE})| = \left| - \frac{\Delta f \cdot v_p}{(\Delta\theta - \sigma_{\Delta\theta}) \cdot f \cdot v_{sI}} \right| \leq 1 \quad (4.6)$$

This calculation is done for each of the measurements and for v_{sI} . The total equivalent "variance", since the measurements are independent, is thus

$$\sigma_{R_{cpaI}}^2 = \sigma_{R_{cpaI}}^2(\Delta\theta) + \sigma_{R_{cpaI}}^2(\Delta f) + \sigma_{R_{cpaI}}^2(v_{sI}) \quad (4.7)$$

Similarly, the equation for X_{cpaI} , equation (3.38), the distance to or from cpa can be used to calculate its equivalent "standard deviation". Since

$$X_{cpaI} = R_o \cdot \cos(\theta_{sI} - \theta_{AVE}) \quad , \quad (4.8)$$

R_o can be substituted from equation (3.38) to give

$$X_{cpaI} = \frac{T \cdot v_{sI}}{\Delta\theta} \cdot \sin(\theta_{sI} - \theta_{AVE}) \cdot \cos(\theta_{sI} - \theta_{AVE}). \quad (4.9)$$

Combining the trigonometric terms yields

$$X_{cpaI} = \frac{T \cdot v_{sI}}{\Delta\theta} \cdot \frac{\sin 2(\theta_{sI} - \theta_{AVE})}{2} \quad (4.10)$$

In this instance the geometric boundary is imposed on the calculation of the angle $\theta_{sI} - \theta_{AVE}$ and restricts X_{cpaI} to be no greater than the maximum range allowed which is $\frac{T \cdot v_{sI}}{\Delta\theta}$.

The equivalent "standard deviation" can be calculated as was done for R_{cpaI} and the equivalent covariance found by an extension of equation (B17). This results in

$$\begin{aligned} \text{cov}[R_{cpaI}; X_{cpaI}] &= \sigma_{R_{cpaI}}(\Delta\theta) \cdot \sigma_{X_{cpaI}}(\Delta\theta) + \\ &\sigma_{R_{cpaI}}(\Delta f) \cdot \sigma_{X_{cpaI}}(\Delta f) + \sigma_{R_{cpaI}}(v_{sI}) \cdot X_{cpaI}(v_{sI}) . \end{aligned} \quad (4.11)$$

The entire initial covariance of estimation error matrix can be filled out in this manner.

To demonstrate the differences in the two error estimates, Table II contains the initial covariance matrix for a computer simulation in which the target was near cpa when the filter was initialized. These values were calculated

using the partial derivative expansion described in Appendix B. The run was made using the $R_{cpa}-X_{cpa}$ filter. In the upper left is the variance of R_{cpaI} . Next on the right is the covariance of R_{cpaI} and X_{cpaI} , then the covariance of R_{cpaI} and v_{sI} , etc.

The diagonal terms, which represent the initial error variances, can be expressed in more convenient units to give

$$\text{var}(R_{cpaI}) = \sigma_{R_{cpaI}}^2 = (14.7 \text{ K yds})^2$$

$$\text{or } \sigma_{R_{cpaI}} = 14.7 \text{ k yds.}$$

$$\text{Similarly } \sigma_{X_{cpaI}} = 14.4 \text{ K yds} ,$$

$$\sigma_{v_{sI}} = 1.68 \text{ yds/sec} ,$$

$$\sigma_{\theta_{sI}} = 71.7^\circ ,$$

$$\text{and } \sigma_{F_{oI}} = 2.5 \text{ Hz.} \quad (4.12)$$

The accuracy of the initial estimates is typically much better than these standard deviations indicate. With the same set of data, the direct calculation described previously was used to calculate the initial covariance of error matrix and the result is presented in Table III. The standard deviations for the diagonal terms are now

216.96	184.20	-5.73	-17.07	33.80
184.20	207.28	-16.43	-17.83	36.37
-5.73	-16.43	2.85	1.21	-2.63
-17.07	-17.83	1.21	1.56	-3.16
33.80	36.37	-2.64	-3.16	6.16

TABLE II. Initial covariance matrix using partial derivative expansion.

78.84	30.34	-0.23	-3.89	7.20
30.34	39.29	-7.84	-3.52	7.36
-0.23	-7.84	2.85	0.60	-1.39
-3.89	-3.52	0.60	0.34	-0.69
7.20	7.36	-1.39	-0.69	1.43

TABLE III. Initial covariance matrix using direct calculation.

$$\sigma_{R_{cpaI}} = 8.78 \text{ K yds} ,$$

$$\sigma_{X_{cpaI}} = 6.27 \text{ K yds} ,$$

$$\sigma_{v_{sI}} = 1.68 \text{ yds/sec} ,$$

$$\sigma_{\theta_{sI}} = 33.5^\circ ,$$

$$\text{and } \sigma_{F_{oI}} = 1.2 \text{ Hz} . \quad (4.13)$$

The net effect of the partial derivative method of covariance matrix calculation is to deemphasize the initial conditions in favor of the first several measurements. The second method applies a filtering weight more in line with the accuracy of each of the initial states.

Not only are the magnitudes of the errors quite different but the orientation of the error ellipses which are determined by the covariance terms are different as well. This is illustrated in Fig. 4.2 for the position variables R_{cpaI} and X_{cpaI} . For the two covariance matrices in Tables II and III a coordinate rotation is applied to give uncorrelated position coordinates. Figure 4.2a shows the orientation to be aligned with the computed values of R_{cpaI} and X_{cpaI} , and not dependent on the sensor position. Figure 4.2b, for the direct calculation, shows the error ellipse oriented with the bearing line from the sensor to the target at the initial position. This

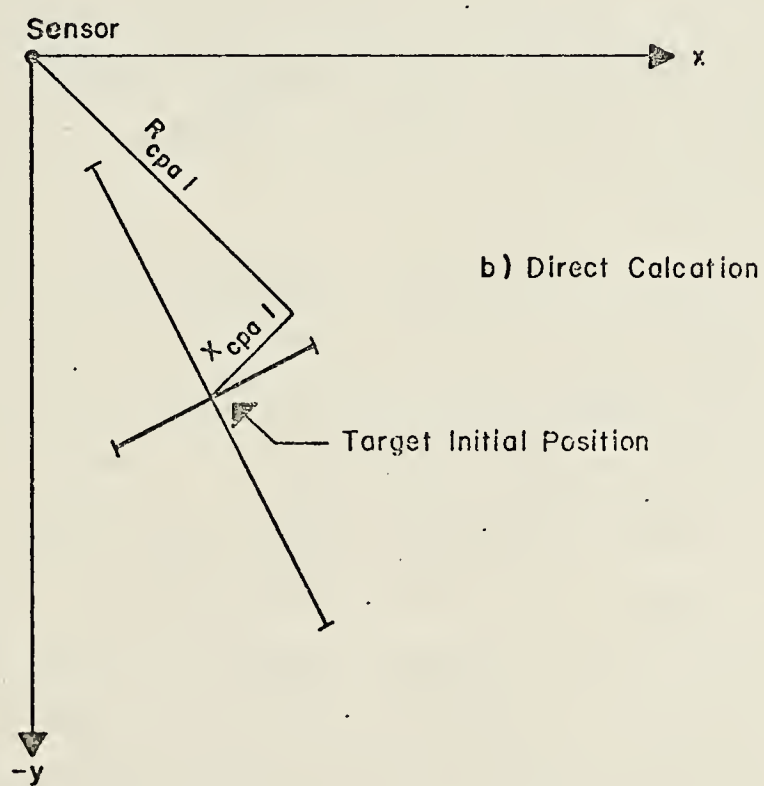
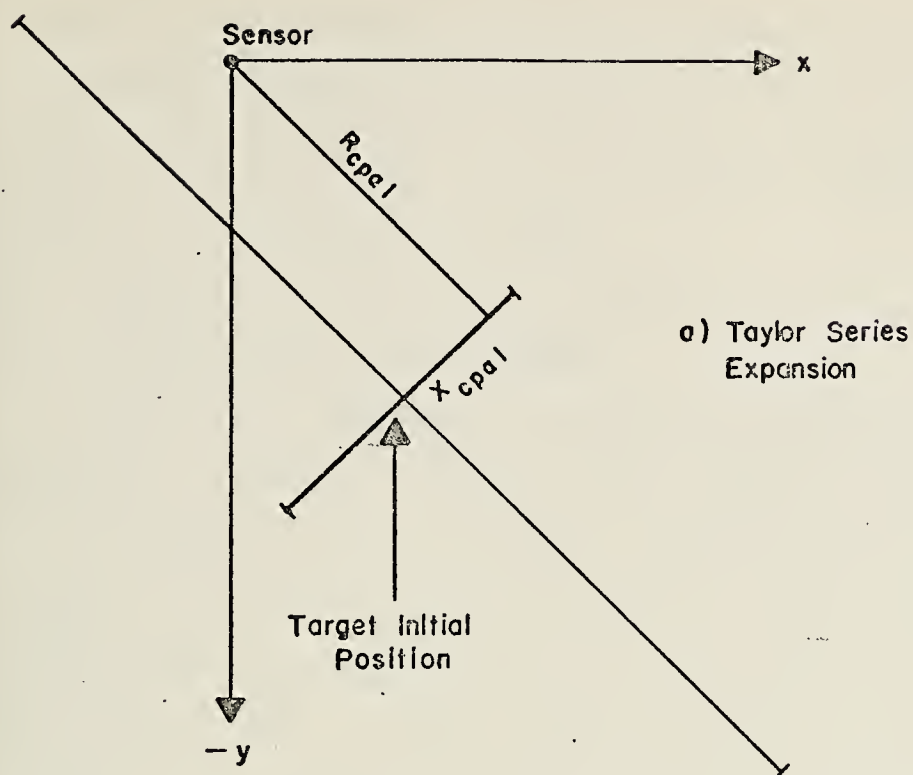


Fig.4.2 Error Ellipses for Initial Covariance Calculation

rotation is believed to occur because of the boundary conditions imposed on the calculated errors and represents a more precise picture of the uncertainty regions associated with the initial state values.

B. GENERAL COVARIANCE UPDATE AND APPLICATION TO KALMAN FILTER EQUATIONS

One of the problems with Extended Kalman filtering is that the linearity assumption used when the nonlinear terms are expanded may not be valid, particularly in the early stages of filtering when the initial filter states are significantly different from the true values. There may be cases where boundary conditions impose constraints on the range of the nonlinear functions which cannot be handled easily using the Extended Kalman approach. In the final analysis the linear expansion will only be valid when the error estimates accurately approximate the true errors in the problem.

To obtain these accurate error estimates, the method developed for the calculation of the initial covariance of estimation error matrix described previously, can be applied. In this development one additional point becomes clear: the partial derivative method (Extended Kalman), requires the linearity condition to be valid for a region in state space extending one standard deviation in all dimensions. When this condition is not satisfied, the direct change method will provide more accurate approximations to the errors.

Consider a discrete problem with n states and m observation variables.

$$\begin{aligned}\underline{x}(k+1) &= \underline{f}(\underline{x}(k)) + \underline{w}(k) \\ \underline{z}(k) &= \underline{h}(\underline{x}(k)) + \underline{v}(k)\end{aligned}\tag{4.14}$$

For an Extended Kalman filter the equations have the form

$$\begin{aligned}\underline{x}(k+1) &= \Phi(k) \underline{x}(k) + \underline{w}(k) \\ \underline{z}(k) &= H(k) \underline{x}(k) + \underline{v}(k)\end{aligned},\tag{4.15}$$

with

$$\Phi(k) = \left. \frac{\partial \underline{f}}{\partial \underline{x}} \right|_{\underline{x}=\hat{\underline{x}}(k)}\tag{4.16}$$

and

$$H(k) = \left. \frac{\partial \underline{h}}{\partial \underline{x}} \right|_{\underline{x}=\underline{x}'(k)}.\tag{4.17}$$

The predicted state covariance matrix is found from

$$P'(k) = \Phi(k) P(k) \Phi(k)^T + Q(k)\tag{4.18}$$

where $P(k)$ is the current covariance of estimation error matrix. To see how the linear expansion affects this calculation, pre-factor and post-factor a diagonal matrix

of the standard deviations of the states out of the covariance matrix. $P(k)$ is given by

$$P(k) = \begin{bmatrix} \sigma_{x_1}^2 & \text{cov}(x_1 x_2) & \dots \\ \text{cov}(x_1 x_2) & \sigma_{x_2}^2 & \\ \vdots & \cdot & \ddots \\ \text{cov}(x_1 x_n) & \dots & \sigma_{x_n}^2 \end{bmatrix} \quad (4.19)$$

Performing the pre- and post-factoring yields

$$P(k) = \begin{bmatrix} \sigma_{x_1} & & & \\ & \sigma_{x_2} & & \\ & & 0 & \\ & & & \sigma_{x_n} \\ 0 & & & & \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{\text{cov}(x_1 x_2)}{\sigma_{x_1} \sigma_{x_2}} & & \frac{\text{cov}(x_1 x_n)}{\sigma_{x_1} \sigma_{x_n}} \\ \frac{\text{cov}(x_1 x_2)}{\sigma_{x_1} \sigma_{x_2}} & 1 & & \\ & & \ddots & \\ \frac{\text{cov}(x_1 x_n)}{\sigma_{x_1} \sigma_{x_n}} & & \dots & 1 \end{bmatrix} \cdot \begin{bmatrix} \sigma_{x_1} & & & \\ & \sigma_{x_2} & & \\ & & 0 & \\ & & & \sigma_{x_3} \\ & & & & \sigma_{x_n} \end{bmatrix} \quad (4.20)$$

$$\text{or} \quad P(k) = \Sigma(k) R(k) \Sigma(k) \quad (4.21)$$

Σ is the diagonal matrix of standard deviations and R is recognized as the matrix of correlation coefficients whose elements are

$$[R]_{ij} = \frac{\text{cov}(x_i, x_j)}{\sigma_{x_i} \sigma_{x_j}} \quad (4.22)$$

If the prediction equation (4.18) is now applied, the result is

$$P'(k) = \Phi(k) \cdot \Sigma(k) \cdot R(k) \cdot \Sigma^T(k) \cdot \Phi^T(k) + Q(k) \quad (4.23)$$

The matrix product $\Phi(k) \cdot \Sigma(k)$ is given by

$$\begin{aligned} \Phi(k)\Sigma(k) &= \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & & & \frac{\partial f_1}{\partial x_n} \\ & \frac{\partial f_2}{\partial x_2} & & \\ & & \ddots & \\ \frac{\partial f_n}{\partial x_1} & & & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{x_1} & & & \\ & \sigma_{x_2} & & \\ & & \ddots & \\ & & & \sigma_{x_n} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\partial f_1}{\partial x_1} \sigma_{x_1} & \frac{\partial f_1}{\partial x_2} \sigma_{x_2} & \dots & \frac{\partial f_1}{\partial x_n} \sigma_{x_n} \\ \vdots & \frac{\partial f_2}{\partial x_2} \sigma_{x_2} & & \vdots \\ \vdots & & \ddots & \\ \frac{\partial f_n}{\partial x_1} \sigma_{x_1} & \dots & \dots & \frac{\partial f_n}{\partial x_n} \sigma_{x_n} \end{bmatrix} \quad (4.24) \end{aligned}$$

The elements of this matrix product, represent the linear expansion of the function $f(\underline{x}(k))$, carried over a one-standard-deviation interval of each of the states. Thus if the linear expansion is not valid over that interval, inaccurate estimates of the predicted covariance could result.

However, if each element in the $\Phi(k)\Sigma(k)$ matrix,

$\frac{\partial f_i(\underline{x})}{\partial x_j} \sigma_{x_j}$ is replaced by

$$\frac{f_i(x_1, x_2, \dots, x_j + \sigma_{x_j}, \dots, x_n) - f_i(x_1, x_2, \dots, x_j - \sigma_{x_j}, \dots, x_n)}{2} \quad (4.25)$$

then, even though the function is nonlinear in the states, the values obtained for use in the prediction equation (4.23), will be a better estimate of the actual errors.

Each of the Extended Kalman filter equations (3.5a) through (3.5f) can be written using the direct change calculation. Let $\tilde{F}(k)$ be the direct one-standard-deviation expansion matrix for the nonlinear state vector function $\underline{f}(\underline{x}(k))$, and $\tilde{H}(k)$ be the direct expansion matrix for $\underline{h}(\underline{x}(k))$. Then the Extended Kalman filter equations are

$$\begin{aligned} P'(k) &= \tilde{F}(k-1)R(k-1)\tilde{F}^T(k-1) + Q(k-1) \\ &= \Sigma'(k) R'(k) \Sigma'(k) \end{aligned} \quad (4.26)$$

$$G(k) = \Sigma'(k)R'(k)\tilde{H}'^T(k)[\tilde{H}(k)R'(k)\tilde{H}^T(k) + R(k)]^{-1} \quad (4.27)$$

$$\begin{aligned} P(k) &= P'(k) - \Sigma'(k)R'(k)\tilde{H}'^T(k) \cdot [\tilde{H}(k)R'(k)\tilde{H}^T(k) + R(k)]^{-1} \\ &\quad \cdot \tilde{H}(k)R'(k)\Sigma'(k) \\ &= [\Sigma'(k) - G(k)\tilde{H}(k)]R'(k)\Sigma'(k) \quad . \end{aligned} \quad (4.28)$$

$\Sigma'(k)$ is the predicted pre-factored standard deviation matrix and $R'(k)$ is the predicted correlation coefficient matrix. The filter equation for $\hat{x}(k)$ and the prediction equation for $x'(k)$ remain the same.

Any geometric or physical state boundaries can be applied directly to the calculation of equivalent "standard deviation", given by equation (4.25), to reduce the range of the function and thus the uncertainty.

While the updated values may not carry with them probability implications, as indeed they do not in the Extended Kalman approach, they would more accurately approximate the errors in the updating calculations due to the error covariances of the states..

V. COMPUTER SIMULATION

To evaluate the two types of filters a simulation program was written which computes the true values of sensor measurements at each sensor for a specific target track, adds independent Gaussian noise values to these measurements, and supplies the results for input to the two filters.

For the extended Kalman filter in the X-Y coordinate system, the provision was made to include up to two additional frequency-only sensors. The Extended Kalman $R_{cpa} - X_{cpa}$ filter has the provision for an additional angle, frequency sensor, as well as up to two additional frequency-only sensors. For the purpose of the simulation the rest frequency of the target was chosen to be 500 Hz. The speed of the target varied between 6 to 12 knots, and a wide variety of headings and geometries were run. These values were chosen in an attempt to provide realistic target parameters to the filters. When using doppler information, the higher the frequency, the more the doppler shift and the better will be the frequency measurement for a given resolution. At 500 Hz, a difference in speed of one knot, provides a shift in frequency of approximately .17 Hz.

A. MONTE CARLO SIMULATION

In order to compare the performance of the two filters, and of each individual filter as various parameters were

changed, Monte Carlo simulation runs were made. The calculated statistics of these simulation runs then were used to contrast filter performance. Each run used input measurements (chosen for convenience to be equally spaced in time) from the simulation program. After each Monte Carlo run, the statistics of the filter states at each time point were calculated. The sample means, \bar{x} , \bar{y} , sample variances, $\text{var}(\hat{x})$, $\text{var}(\hat{y})$, and sample covariances $\text{cov}(\hat{x}, \hat{y})$ for the position variables were computed. Let the position outputs of the filter at time $t(k)$ for a particular run be $\hat{x}_i(k)$ and $\hat{y}_i(k)$. The sample statistics for N runs are¹

$$\bar{x}(k) = \frac{1}{N} \sum_{i=1}^N \hat{x}_i(k) \quad (5.1)$$

$$\bar{y}(k) = \frac{1}{N} \sum_{i=1}^N \hat{y}_i(k) \quad (5.2)$$

$$\text{var}[\hat{x}(k)] = \sigma_x^2 = \frac{1}{N-1} \left[\sum_{i=1}^N \hat{x}_i(k)^2 - N\bar{x}(k)^2 \right] \quad (5.3)$$

$$\text{var}[\hat{y}(k)] = \sigma_y^2 = \frac{1}{N-1} \left[\sum_{i=1}^N \hat{y}_i(k)^2 - N\bar{y}(k)^2 \right] \quad (5.4)$$

$$\text{cov}[\hat{x}(k)\hat{y}(k)] = r_{xy}\sigma_x\sigma_y = \frac{1}{N-1} \left[\sum_{i=1}^N \hat{x}_i(k)\hat{y}_i(k) - N\bar{x}(k)\bar{y}(k) \right] \quad (5.5)$$

¹Deutsch [16] gives the equation for the sample variance which can be readily extended for the sample covariance.

The choice of how many runs to use in a Monte Carlo simulation is always a difficult one and must be a compromise between the computer time for the simulation program and the accuracy of the computed statistics. All the results used for filter comparison in this work represent 200 Monte Carlo runs. Bucy [11] provides a means for estimating the accuracy of the statistics calculated from Monte Carlo runs and his findings are summarized in Appendix D and the results given below for the \bar{x} variable.

For 200 runs the probability that the magnitude of the difference between the specified mean, μ , and the sample mean will be less than ten percent of one standard deviation is .84. That is

$$\Pr \{ |\bar{x} - \mu| < .1 \text{ var}(x) \} = .84 \quad . \quad (5.6)$$

The probability that the specified standard deviation will be within .1 of the sample standard deviation is approximately .95. That is

$$\Pr \{ .9\sigma_x < \sigma < 1.1\sigma_x \} \approx .95 \quad (5.7)$$

with σ the specified standard deviation.

B. ROTATED ERROR COORDINATE SYSTEM

The position variances and covariances calculated by the above technique indicates that the errors in the X and

Y position are highly correlated. Correlation coefficients of .8 or .9 are typical at each time point.

If the errors were normally distributed this would indicate that there exists a rotated coordinate system such that in the new system the orthogonal position components are uncorrelated. This is equivalent to taking the exponent of the joint normal probability density function, and applying a coordinate transformation which eliminates the cross terms.

The exponent (for zero-mean random variables) is

$$\frac{x^2}{\sigma_x^2} - \frac{2r_{xy}xy}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y^2} = C^2 \quad (5.8)$$

When set equal to a constant, this curve, which is an ellipse, is a curve of constant probability. This ellipse does not have its major and minor axes aligned with the coordinate system however. By applying the transformation

$$x' = x \cos \theta + y \sin \theta \quad (5.9a)$$

and

$$y' = y \cos \theta - x \sin \theta \quad (5.9b)$$

with

$$\theta = \frac{1}{2} \tan^{-1} \left[\frac{2 \operatorname{cov}(x,y)}{\sigma_x^2 - \sigma_y^2} \right] , \quad (5.10)$$

the ellipse will be aligned with the x' , y' axis and the resultant random variables will be uncorrelated. The new variances in this system are calculated by

$$\sigma_{x'}^2 = \frac{\sigma_x^2 + \sigma_y^2}{2} + \frac{\operatorname{cov}(xy)}{\sin 2\theta} , \quad (5.11a)$$

and

$$\sigma_{y'}^2 = \frac{\sigma_x^2 + \sigma_y^2}{2} - \frac{\operatorname{cov}(xy)}{\sin 2\theta} . \quad (5.11b)$$

This method of viewing the sample statistics provides insight into the filter performance because it shows the regions of high probability and their relationships to the sensor. The values of $\sigma_{x'}^2$ and $\sigma_{y'}^2$ are referred to as the rotated error variances in the following section.

C. FILTER PERFORMANCE

There are almost an infinite number of variables which can be changed, and their effect on the filter response studied. One of the objectives of this research was to produce a workable filter for the passive location and tracking problem, and this will be demonstrated through the comparisons below.

Each figure is a computer generated Calcomp plot of a simulated track. Unless otherwise stated, a single sensor is positioned at the origin of the coordinate system. The line drawn is the true track with the tic marks indicating the observation times, and the arrow indicating target heading. The asterisks are the Monte Carlo filter points and are the sample average values of position at each time point for the 200 Monte Carlo runs. Some of the figures also show the orientation of the final sample position error ellipse by showing the one standard deviation major and minor axes.

In all of the simulation runs, the X-Y filter gave lower final error variances when compared with the $R_{cpa} - X_{cpa}$ filter. For this reason most of the simulation tests were run with the X-Y filter.

1. Time Between Samples

This requirement will be dictated ultimately by the computational capability of the computer which would be used for an actual system. With the resolution used in the simulation set at .04 Hz, at least a 25-second time record is needed for processing by an FFT processor. The time between samples must be short enough to allow the filter to correct errors, especially in the initial phases. If the time is too long the result could be filter divergence or a poor estimate of the states. This is shown in Fig. 5.1 and 5.2. The first graph is run with 200 seconds between samples and

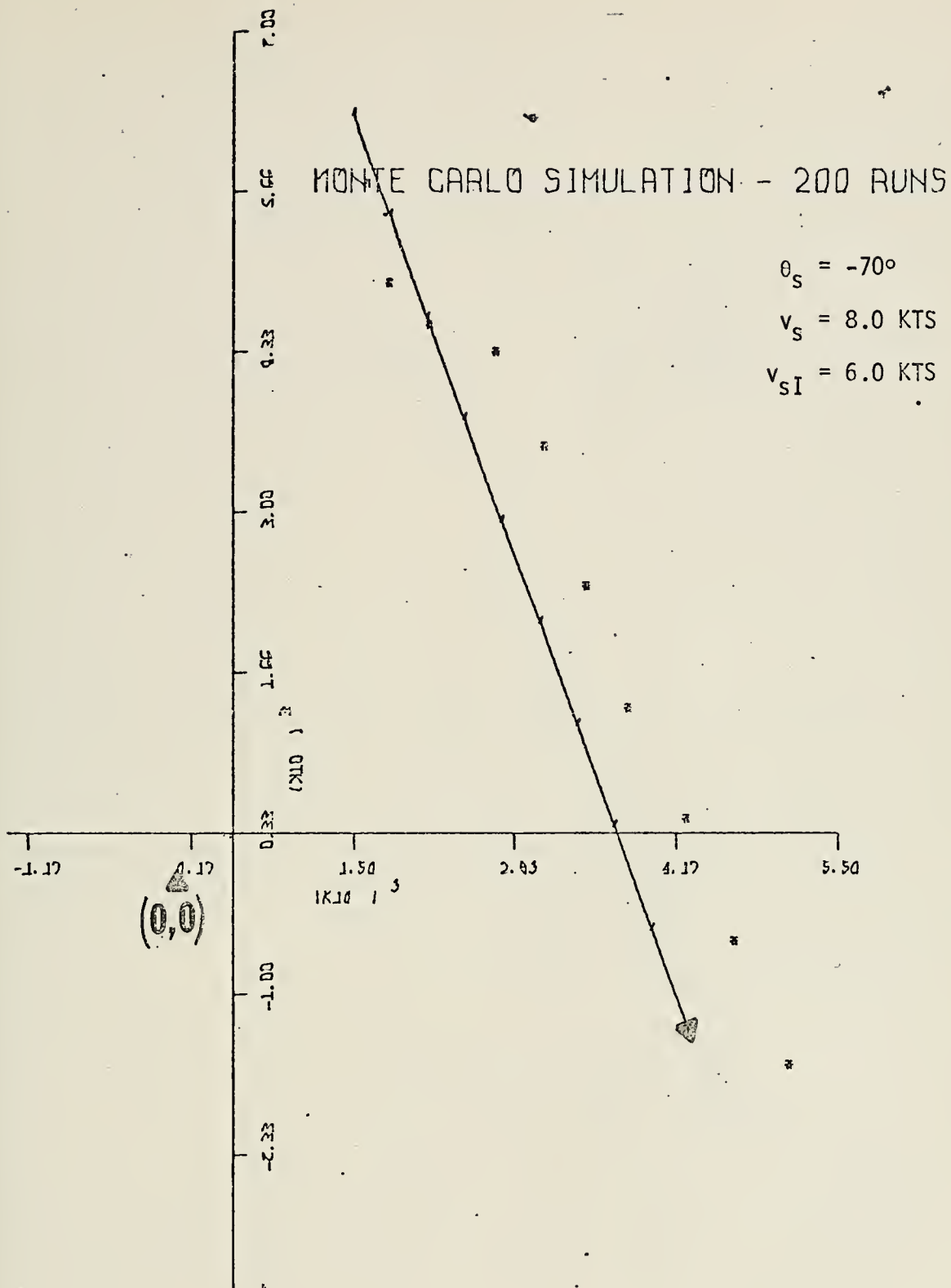


Fig. 5.1 Filter Output and Target Track —
 200 Seconds between Observations

MONTE CARLO SIMULATION - 200 RUNS

$$\theta_s = -70^\circ$$

$$v_s = 8 \text{ KTS}$$

$$v_{sI} = 6 \text{ KTS}$$

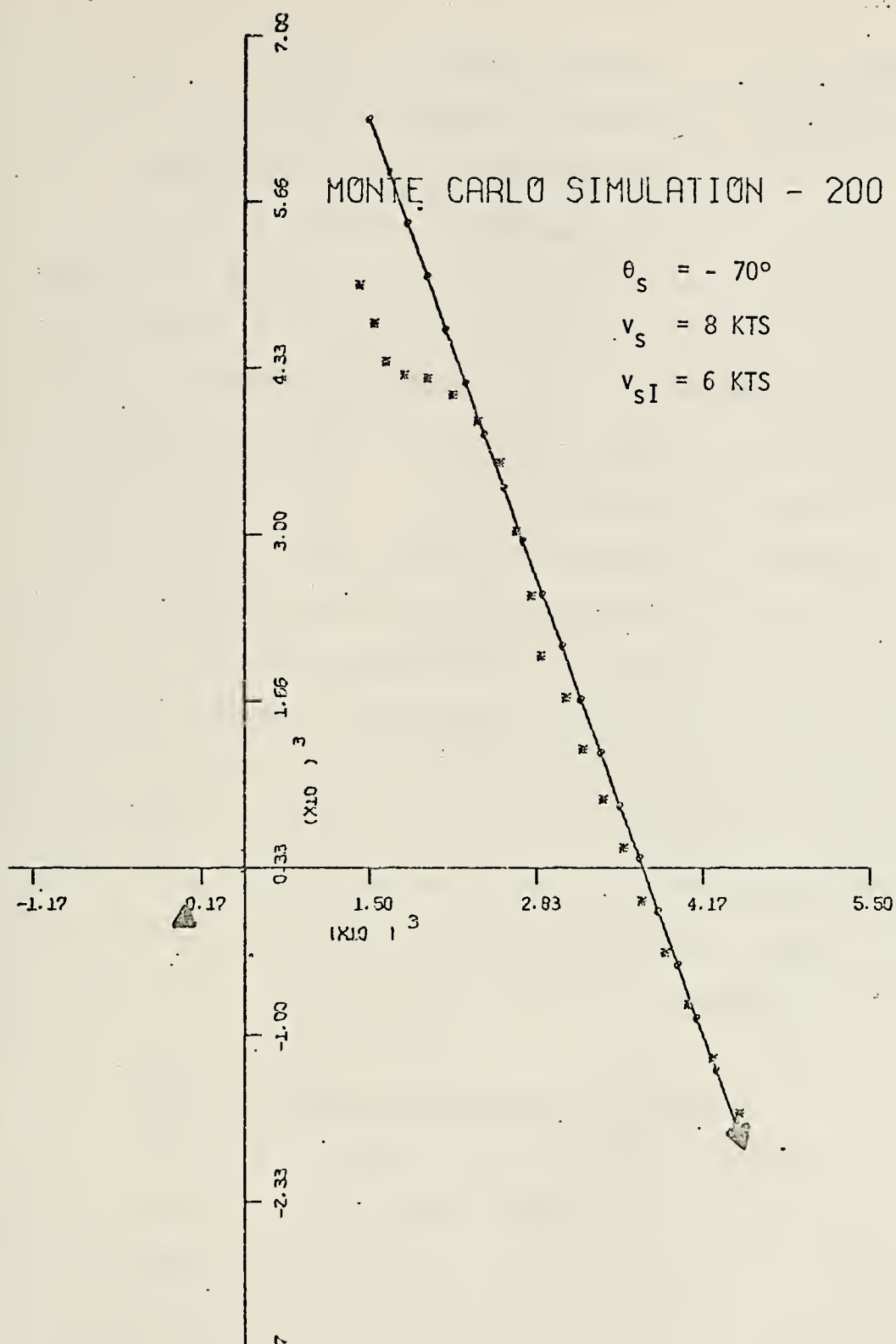


Fig. 5.2 Filter Output and Target Track —
100 Seconds Between Observations

second with 100 seconds between samples. All subsequent simulation runs were made with 100 seconds between samples.

2. Effect of Initial Covariance Matrix

The full initial covariance matrix, as described in Chapter IV, gives as complete a picture as is possible on the probabilistic relationship among the initial states. For this reason it provided better filter transient response, and resulted in lower covariance of error values at the end of a given run. Fig. 5.3 is the response of the X-Y filter when only the diagonal terms of the initial covariance were used - a common practice in implementing filters. Fig. 5.4 is the X-Y filter response for the identical set of measurement data with the full initial covariance implemented. Fig. 5.5 and 5.6, Fig. 5.7 and 5.8 and Fig. 5.9 and 5.10 show similar behavior. The final rotated error ellipses are shown on the plots. In each case the variances were less with the full Matrix than with only diagonal terms. Figures 5.11 and 5.12 and Fig. 5.13 and 5.14 show similar behavior for the $R_{cpa} - X_{cpa}$ filter.

3. Error Ellipses and Number of Sensors

Figure 5.15 shows the X-Y filter response for a single sensor at the origin. Figure 5.16 is a diagram of the one sigma regions for this filter when the rotated error coordinate technique is used. Note how the rotated system tends to align itself with the bearing angle to the filter point. The rotated error ellipses of the velocity components

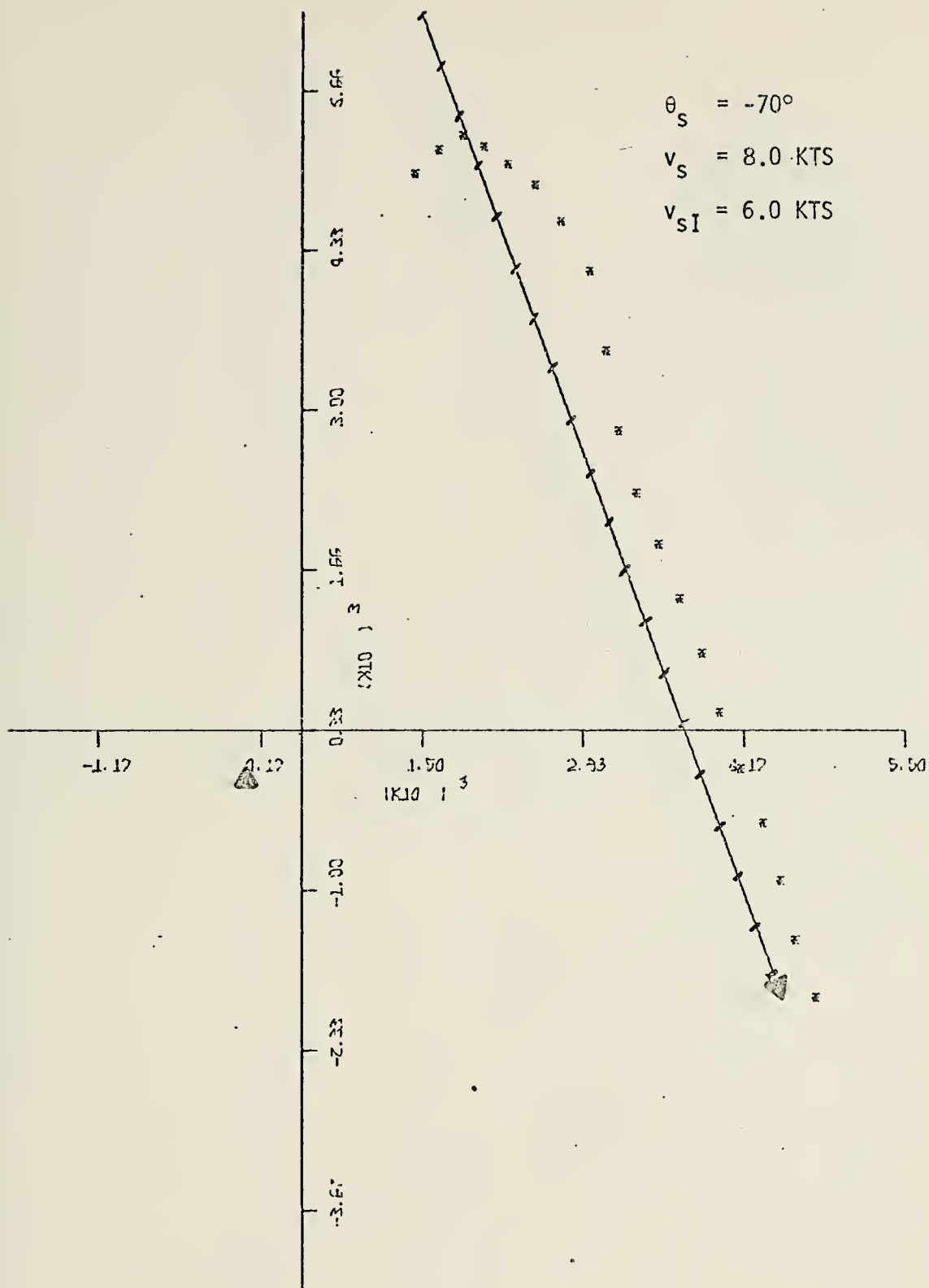


Fig. 5.3 X-Y Filter and Target Track —
Diagonal-Only Initial Covariance Matrix

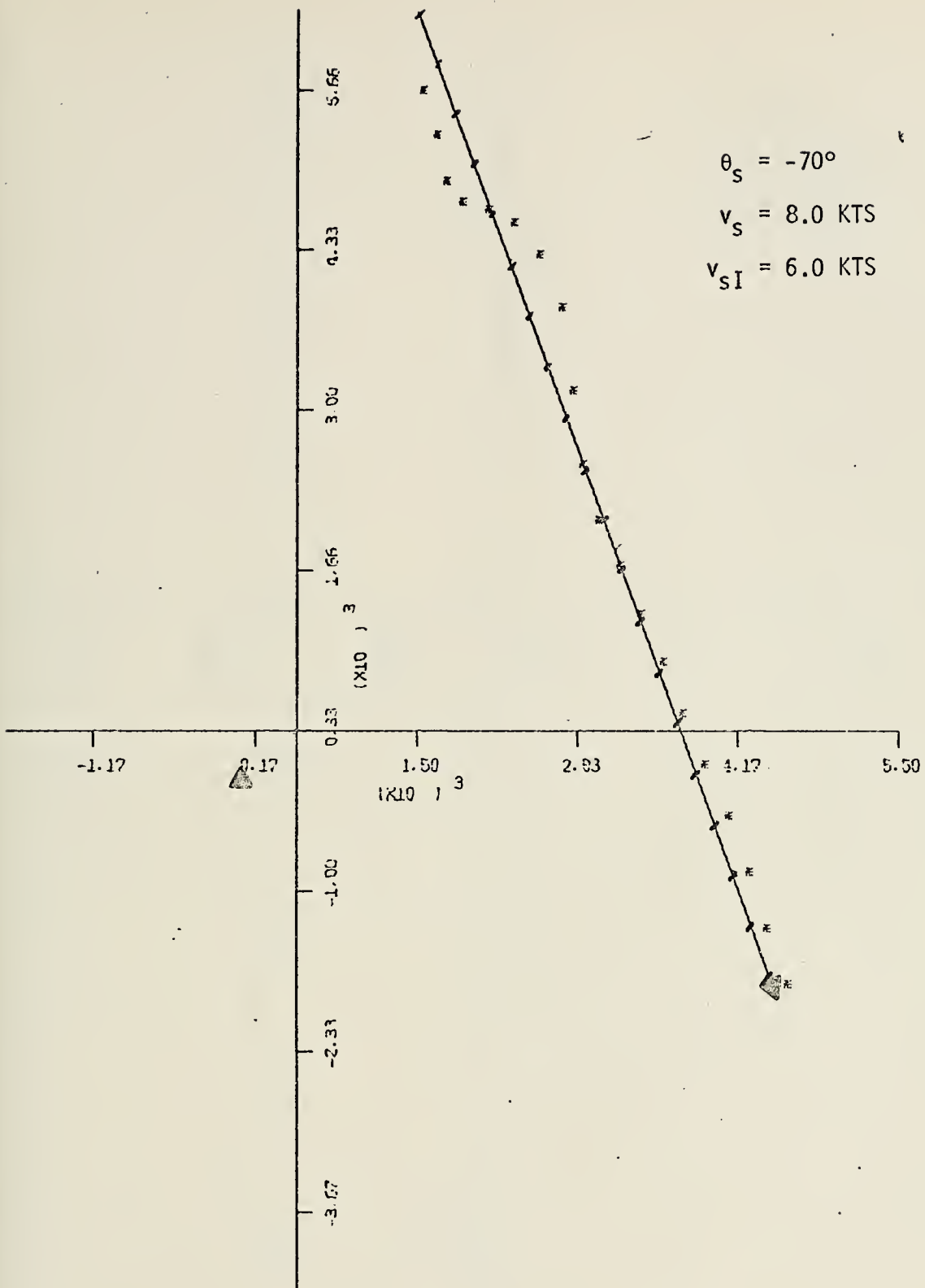


Fig. 5.4 X-Y Filter and Target Track — Full Initial Covariance Matrix

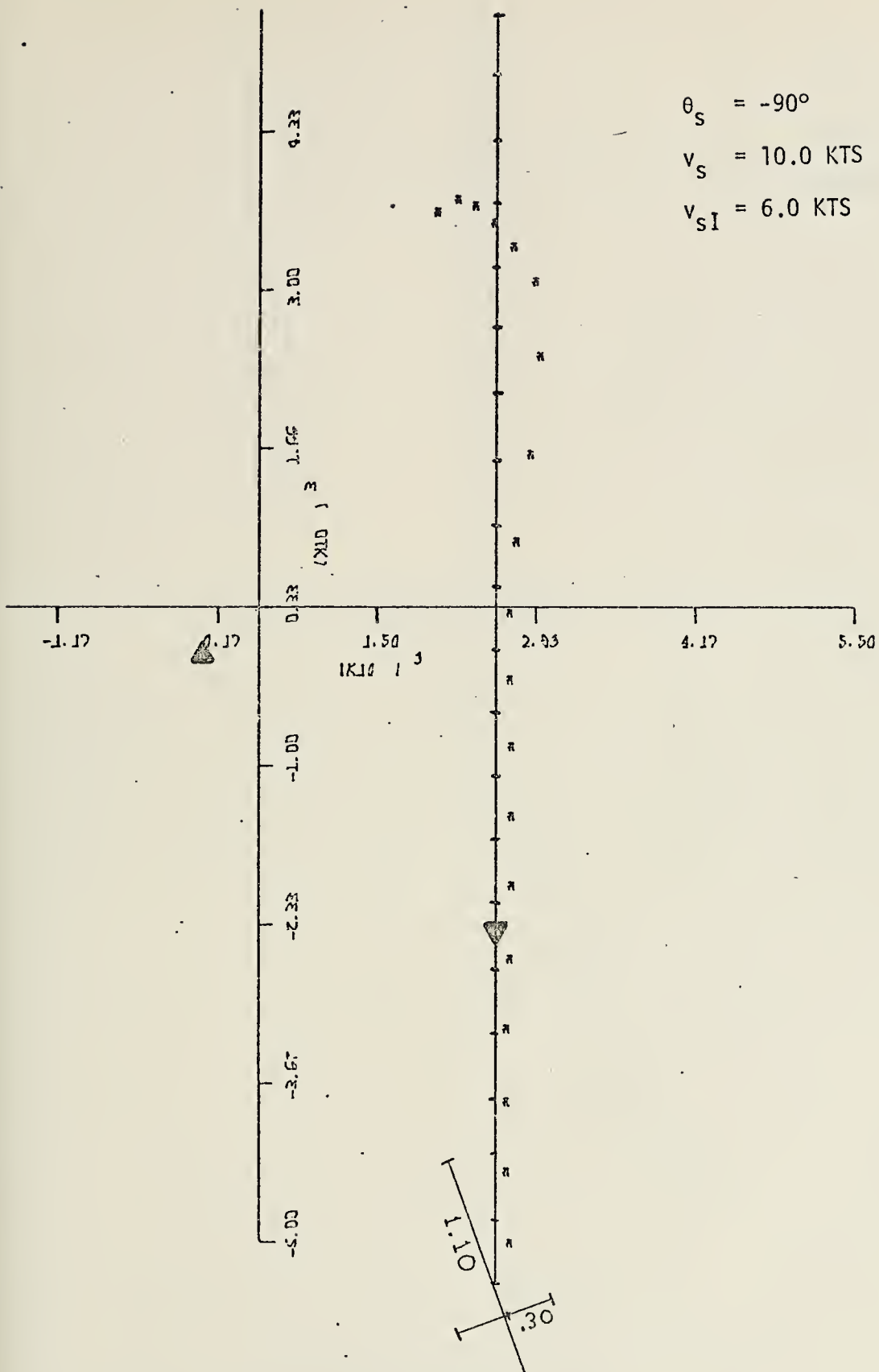


Fig. 5.5 X-Y Filter and Target Track —
Diagonal-Only Initial Covariance Matrix

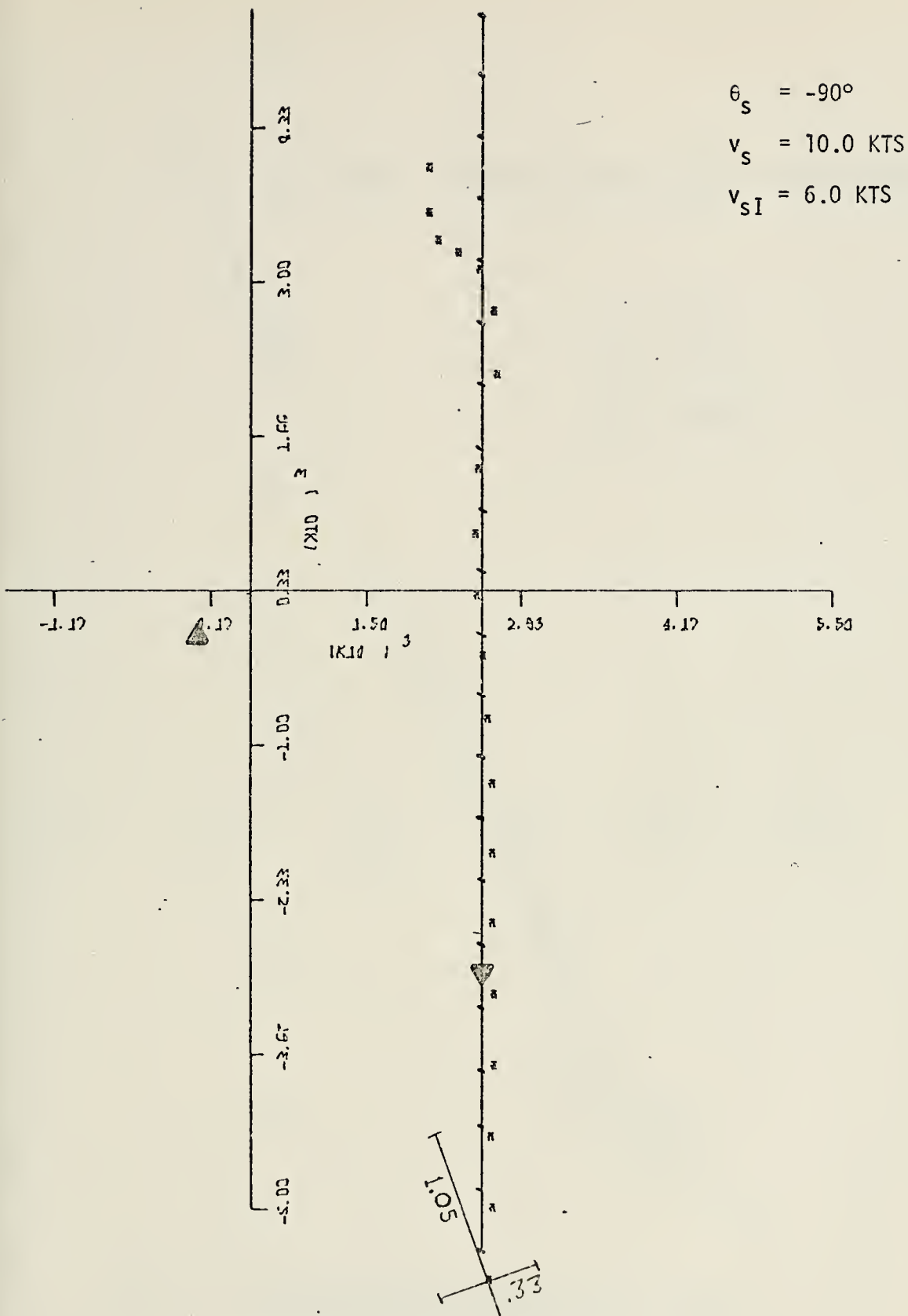


Fig. 5.6 X-Y Filter and Target Track — Full Initial Covariance Matrix

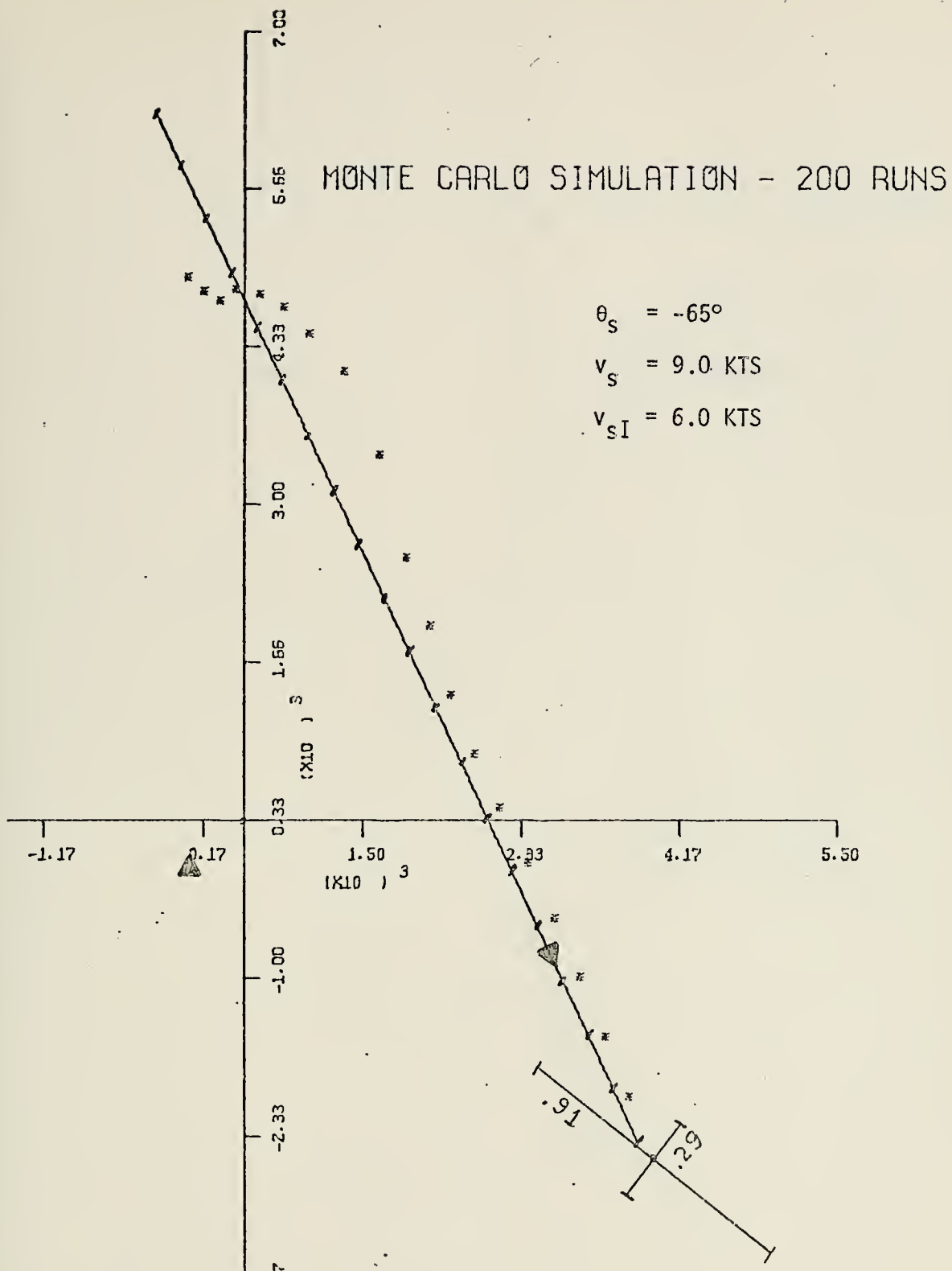


Fig. 5.7 Filter and Target Track —
Diagonal-Only Initial Covariance Matrix

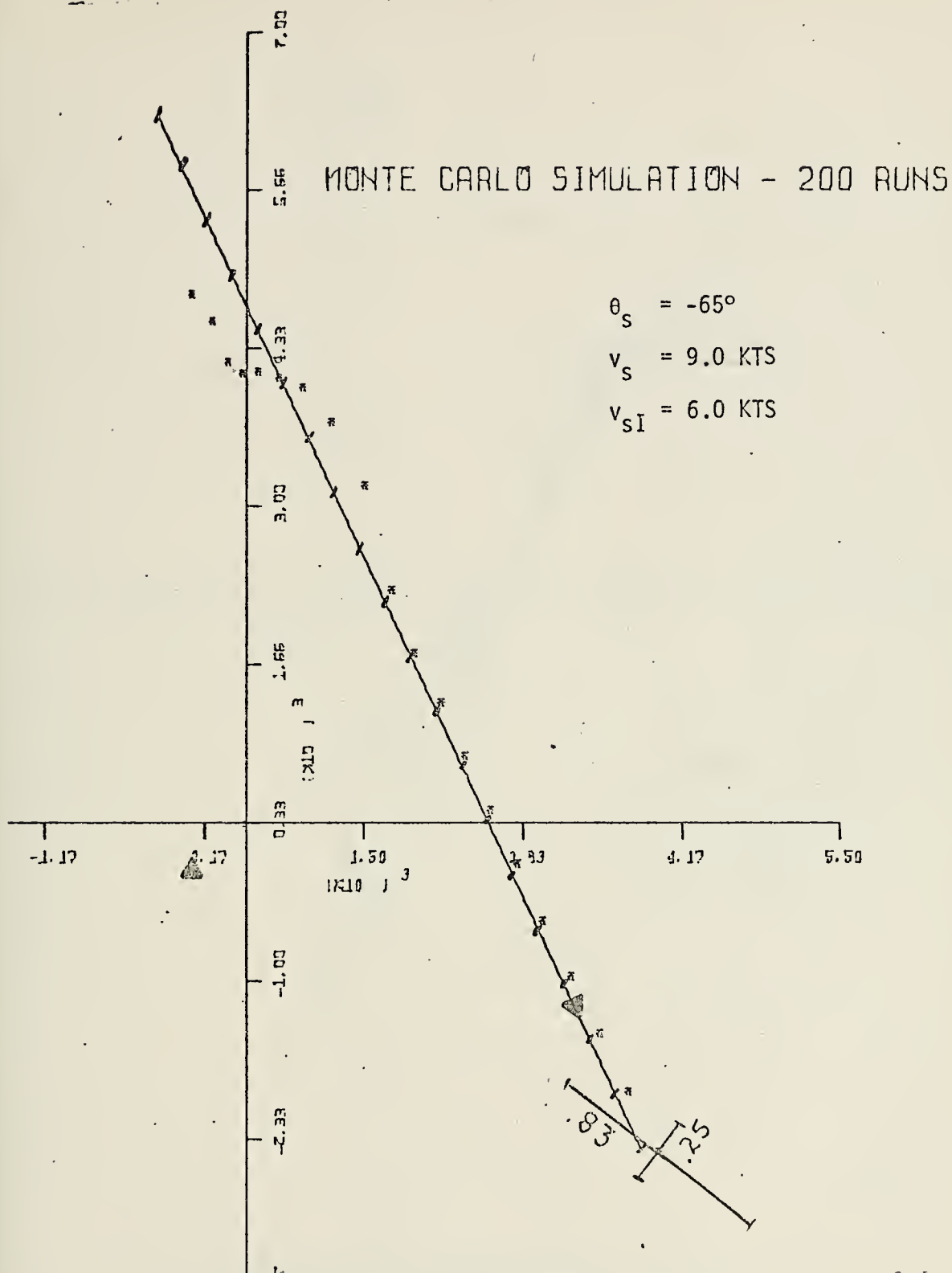


Fig. 5.8 X-Y Filter and Target Track — Full Initial Covariance Matrix

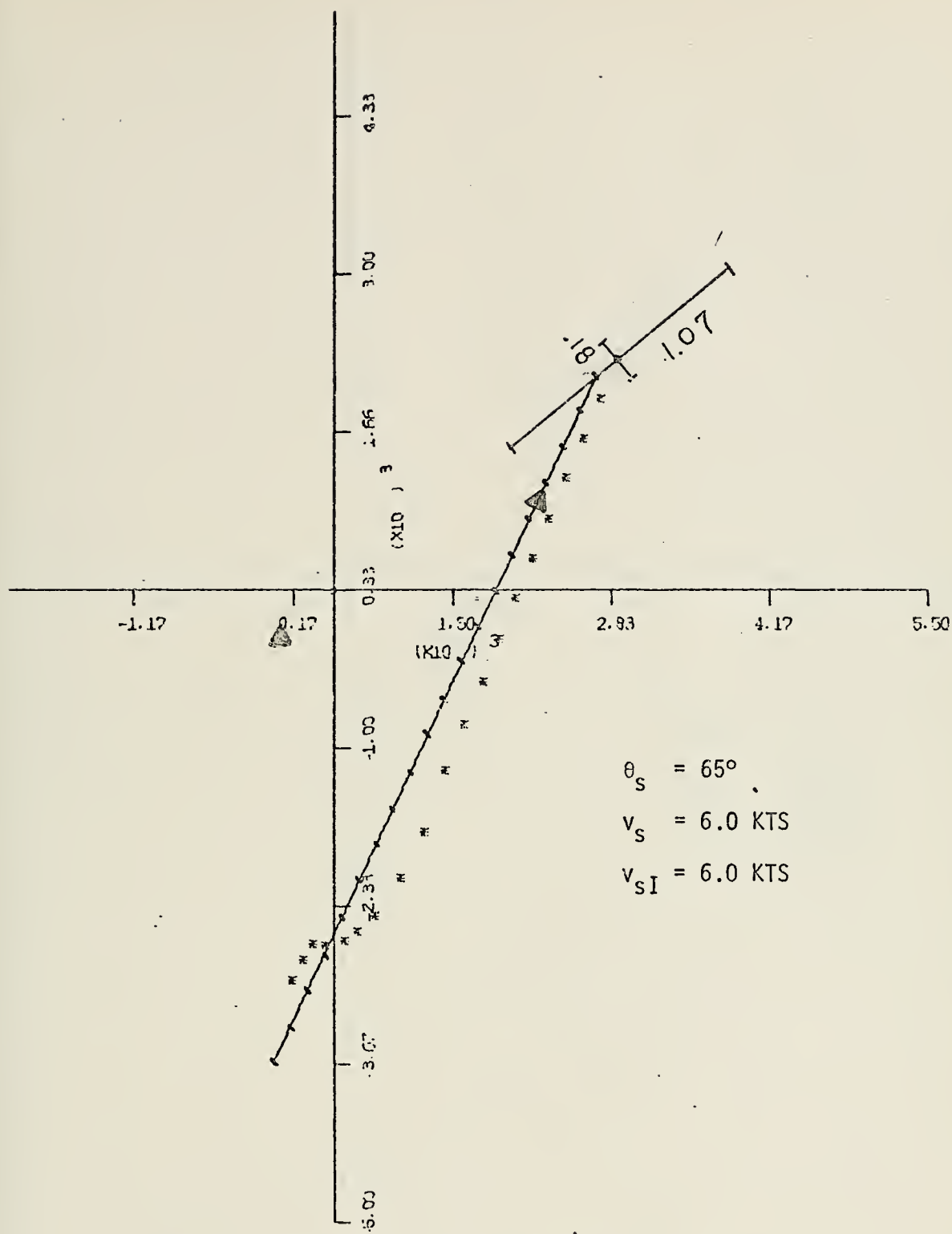


Fig. 5.9 X-Y Filter and Target Track —
Diagonal-Only Initial Covariance Matrix

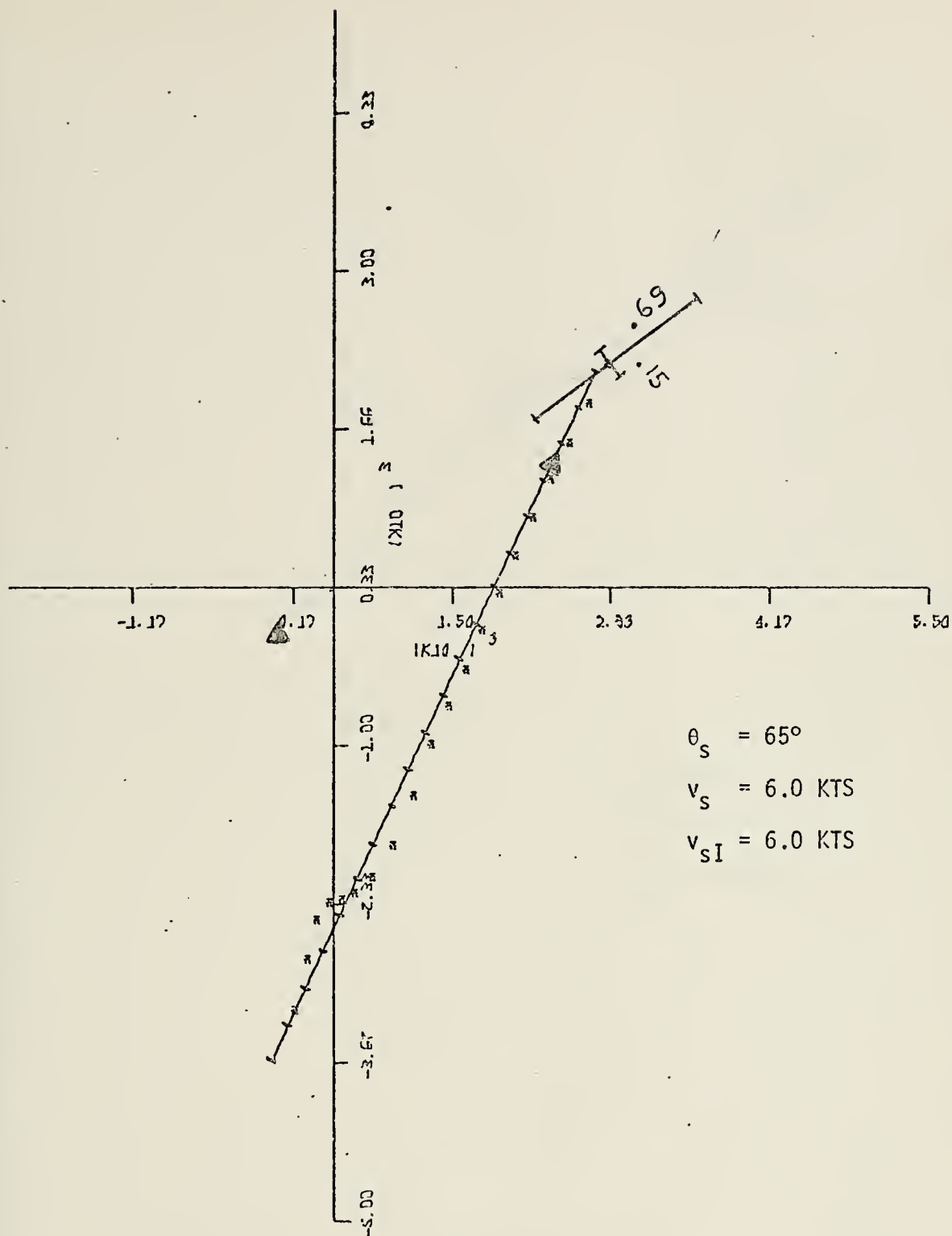


Fig. 5.10 X-Y Filter and Target Track — Full Initial Covariance

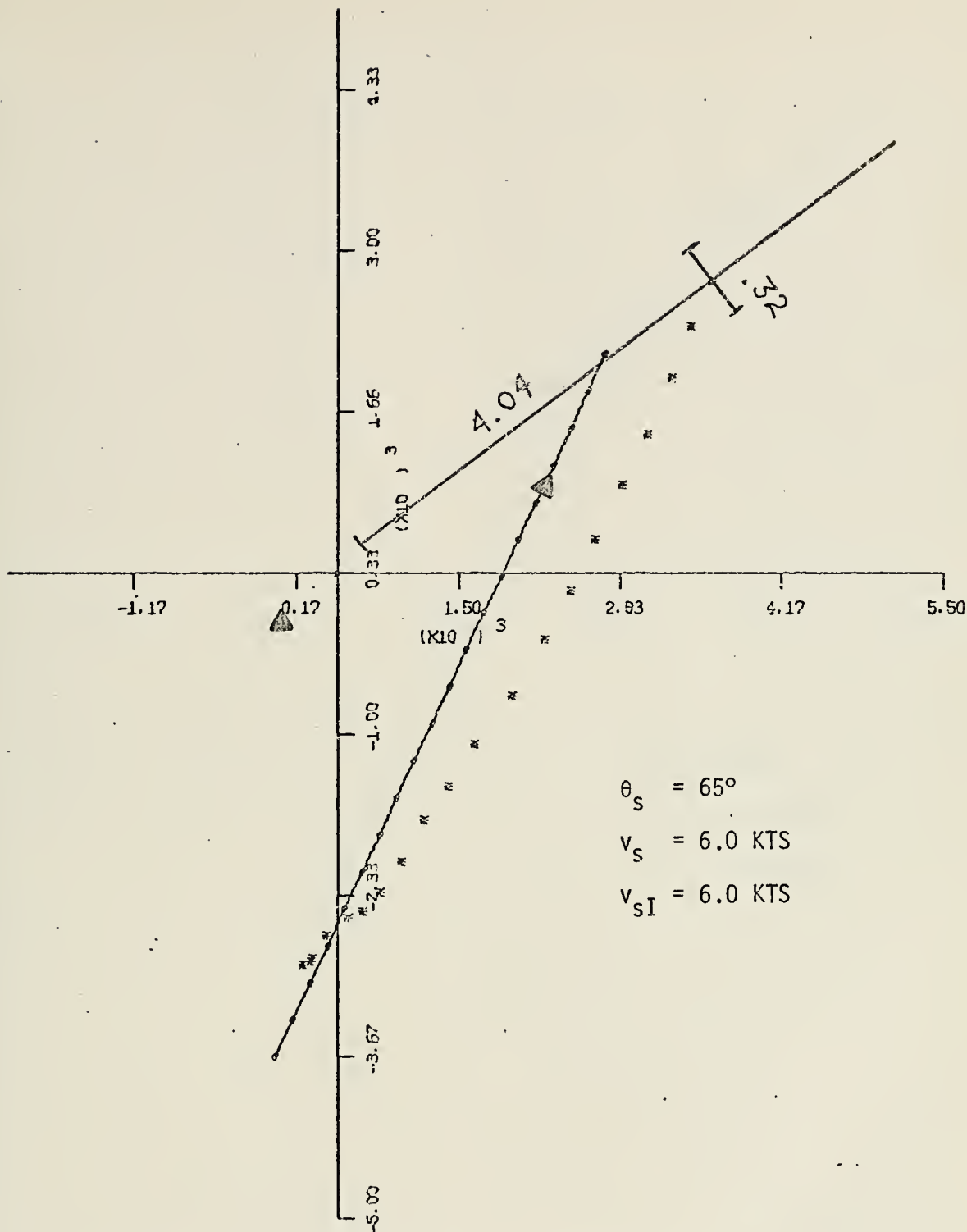


Fig. 5.11 $R_{cpa} - X_{cpa}$ Filter and Target Track —
Diagonal-Only Initial Covariance Matrix

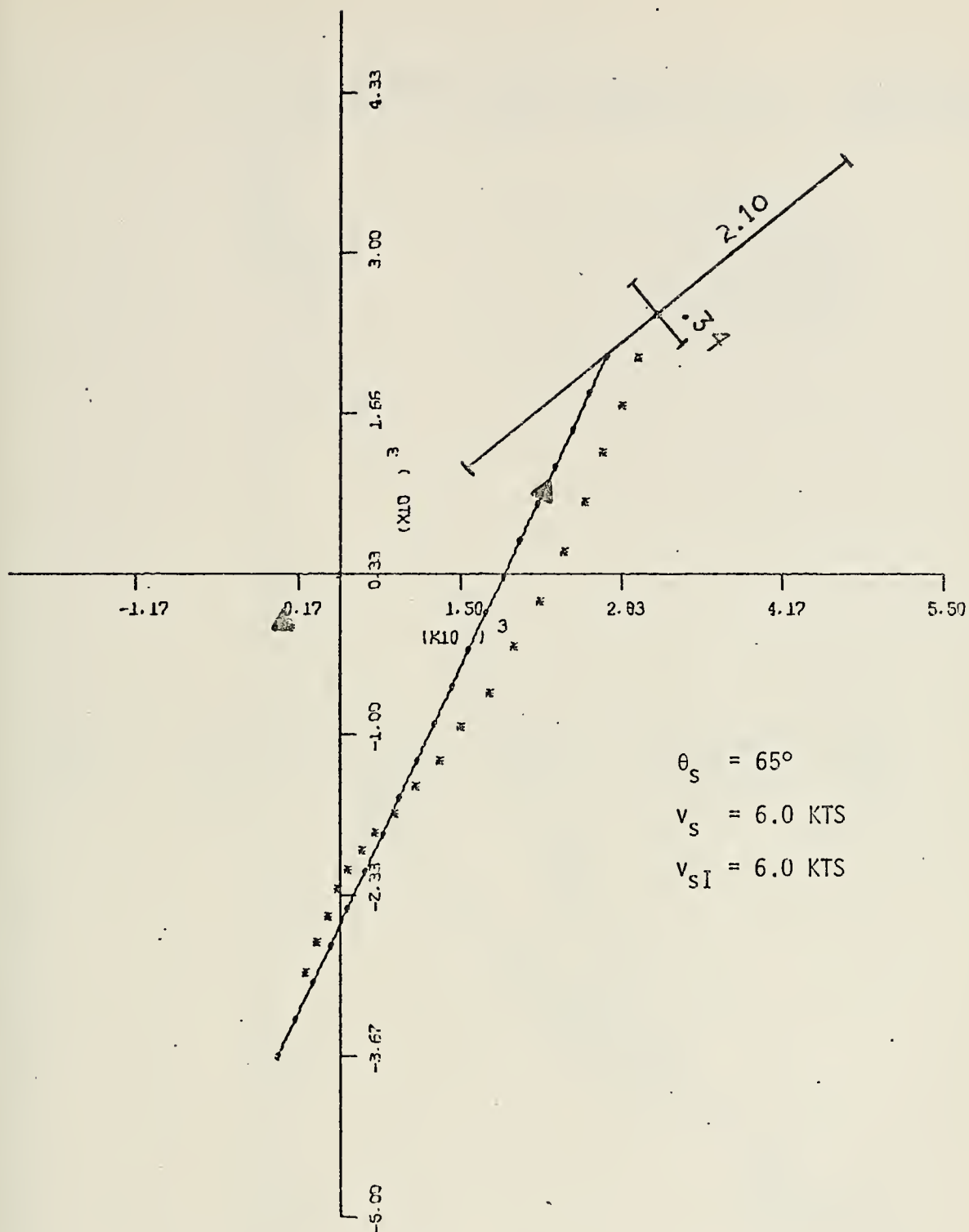


Fig. 5.12 $R_{cpa} - X_{cpa}$ Filter and Target Track —
Full Initial Covariance Matrix

MONTE CARLO SIMULATION - 200 RUNS

$$\theta_s = -65^\circ$$

$$v_s = 9.0 \text{ KTS}$$

$$v_{sI} = 6.0 \text{ KTS}$$

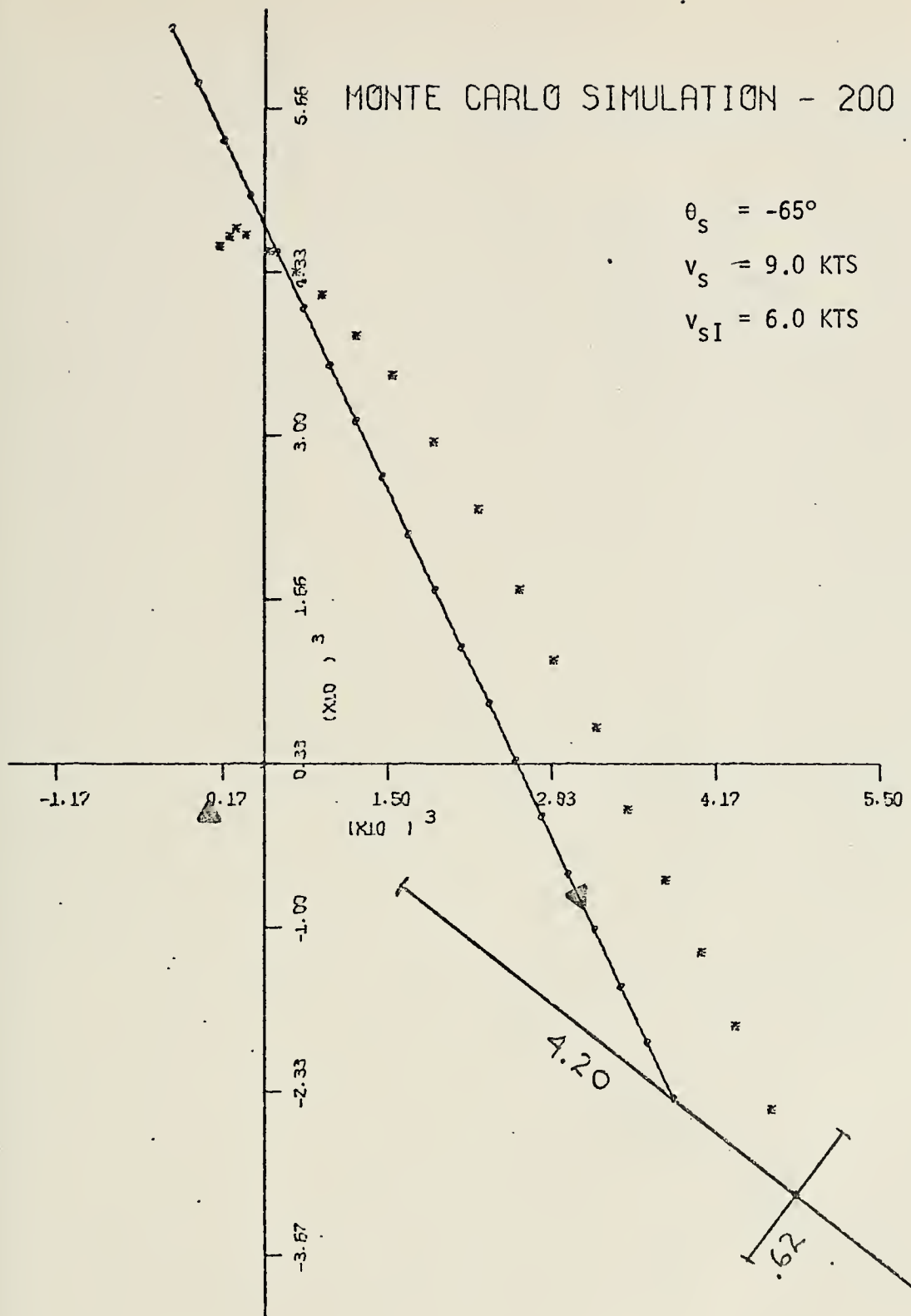


Fig. 5.13 $R_{cpa} - X_{cpa}$ Filter and Target Track —
Diagonal-Only Initial Covariance Matrix

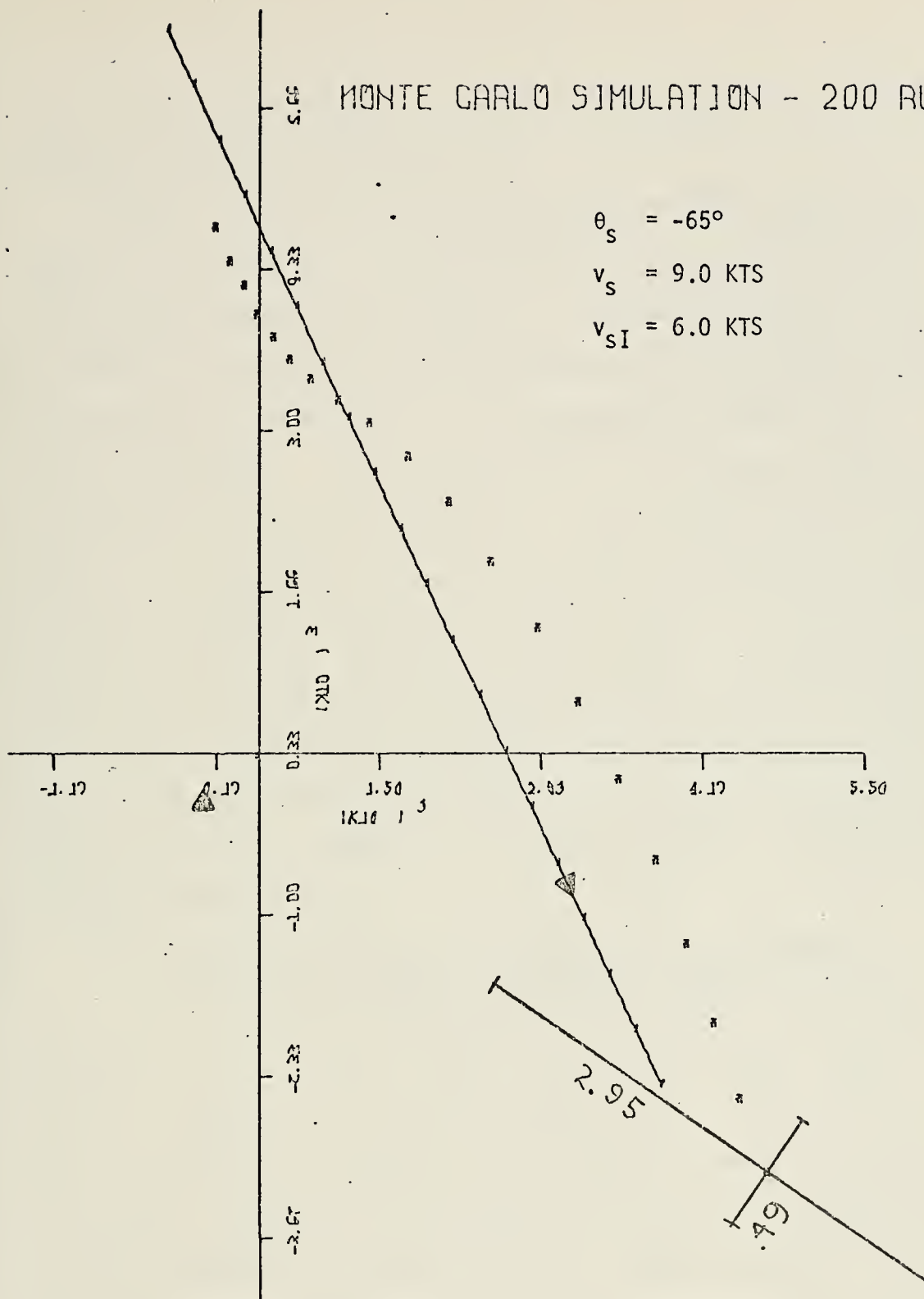


Fig. 5.14 R_{cpa} - X_{cpa} Filter and Target Track —
Full Initial Covariance Matrix

(not shown) tend to show a similar although not as striking alignment with the bearing line.

In Fig. 5.17 an additional frequency-only sensor was added at the position indicated and as expected, improved the averaged filter performance. The rotated variance plot is shown in Fig. 5.18 and the presence of the second sensor has reduced the variance by a significant amount in the radial direction. This result will, of course, depend on the sensors and target geometry. For use with this type of filter, an additional sensor will enhance performance when it sees a definite change in the frequency it is measuring. If the sensor were put near the predicted track for example, very little change in the doppler shifted frequency would take place (except when the target went thru CPA) and the changes which the sensor would measure would be mostly due to the frequency noise.

4. Comparison of the X-Y and $R_{cpa} - X_{cpa}$ Filter

As mentioned previously the X-Y filter performed better in all simulation runs than did the $R_{cpa} - X_{cpa}$ filter. The Monte Carlo tracks were closer to the true track and the final rotated error variances were lower in all cases. Figures 5.19 through 5.24 show a comparison of the two filters for three particular target tracks. In each case the same simulated input measurements were provided to the X-Y and $R_{cpa} - X_{cpa}$ filters. The same initial conditions of position, velocity and rest frequency were

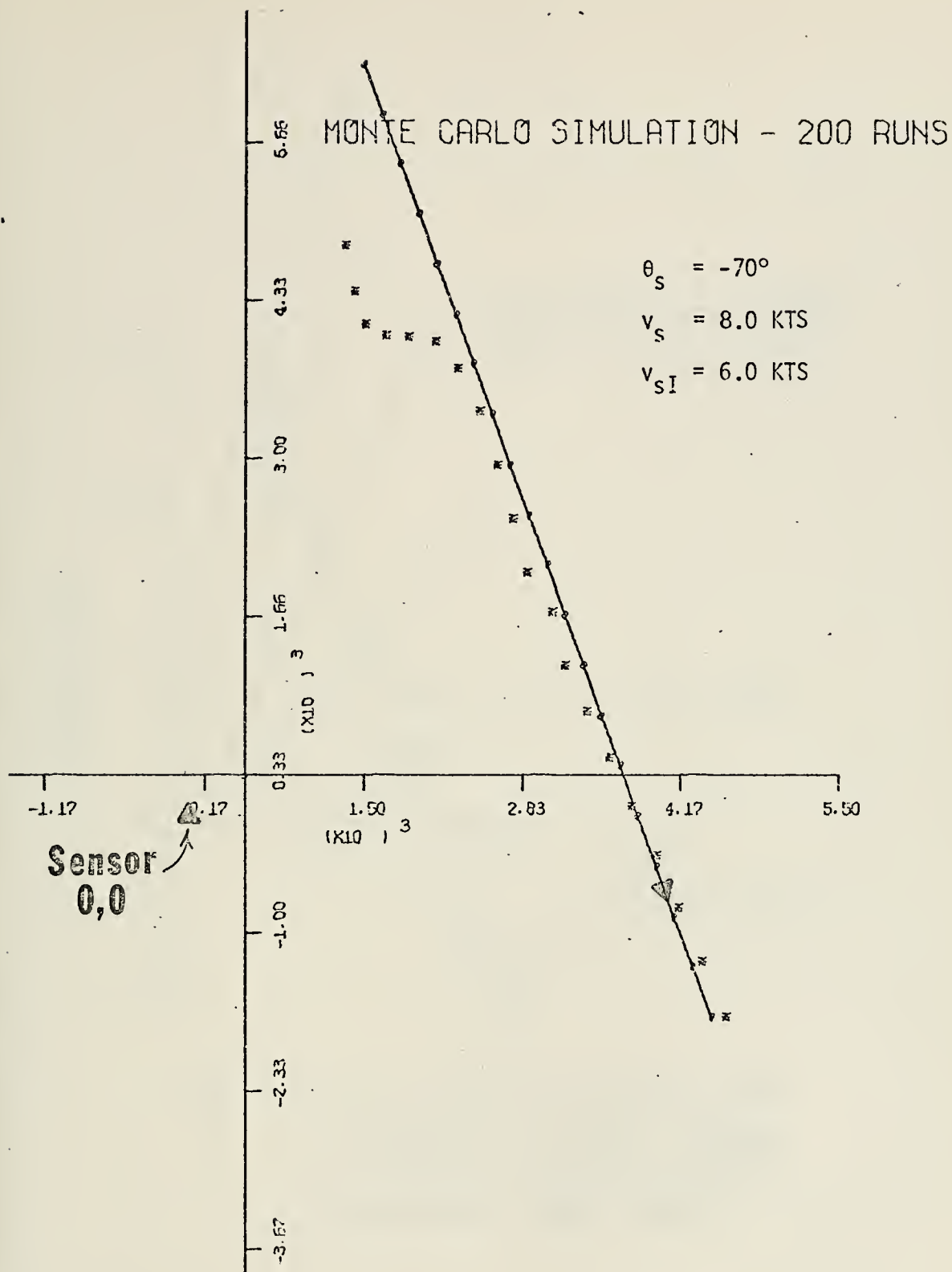


Fig. 5.15 X-Y Filter and Target Track —
Single Sensor of Angle and Frequency at (0,0)

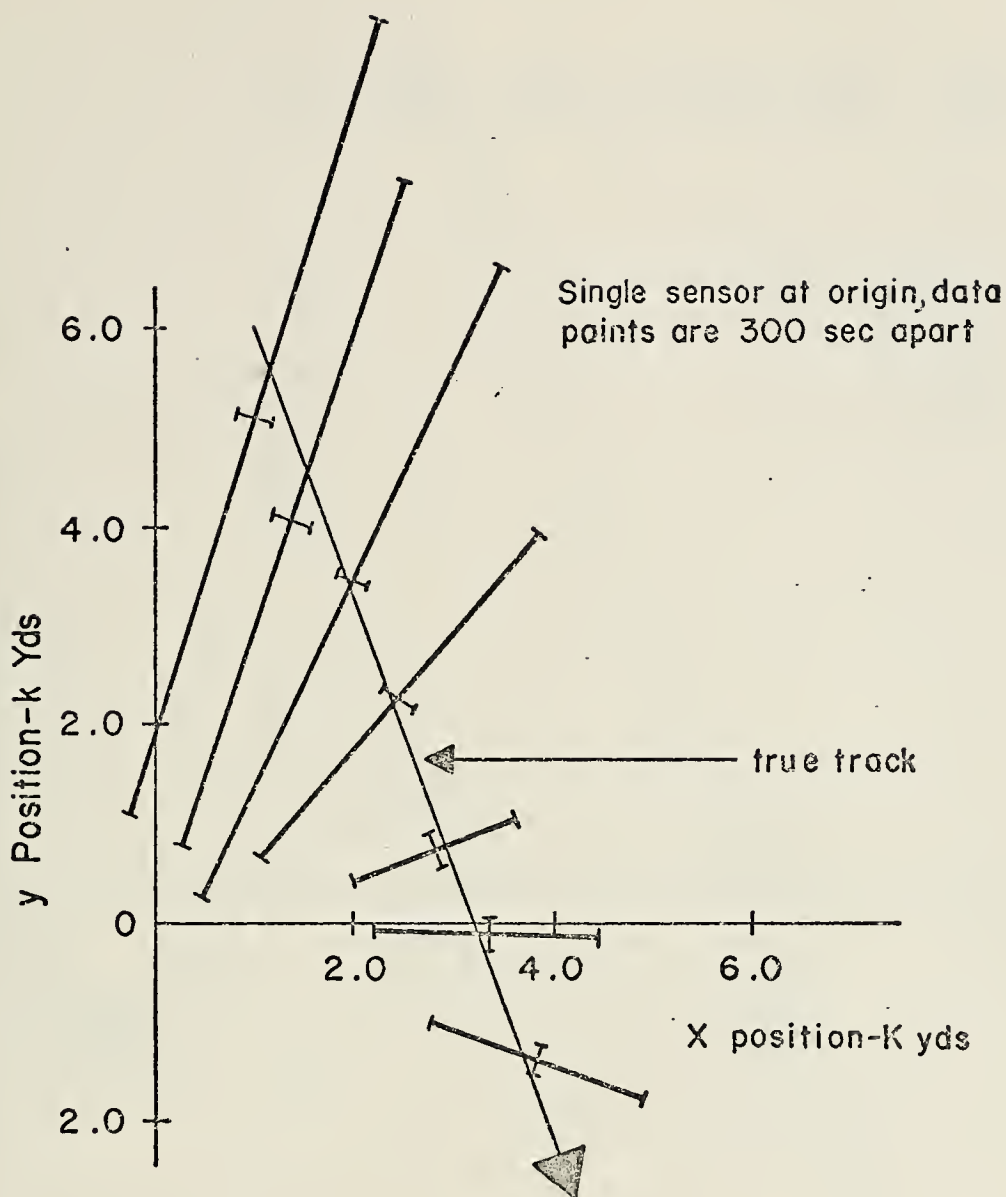


Fig. 5.16 Plot of Major and Minor Axes of Rotated Covariance Ellipses for Position. One-Sigma Points are Marked. Single Sensor

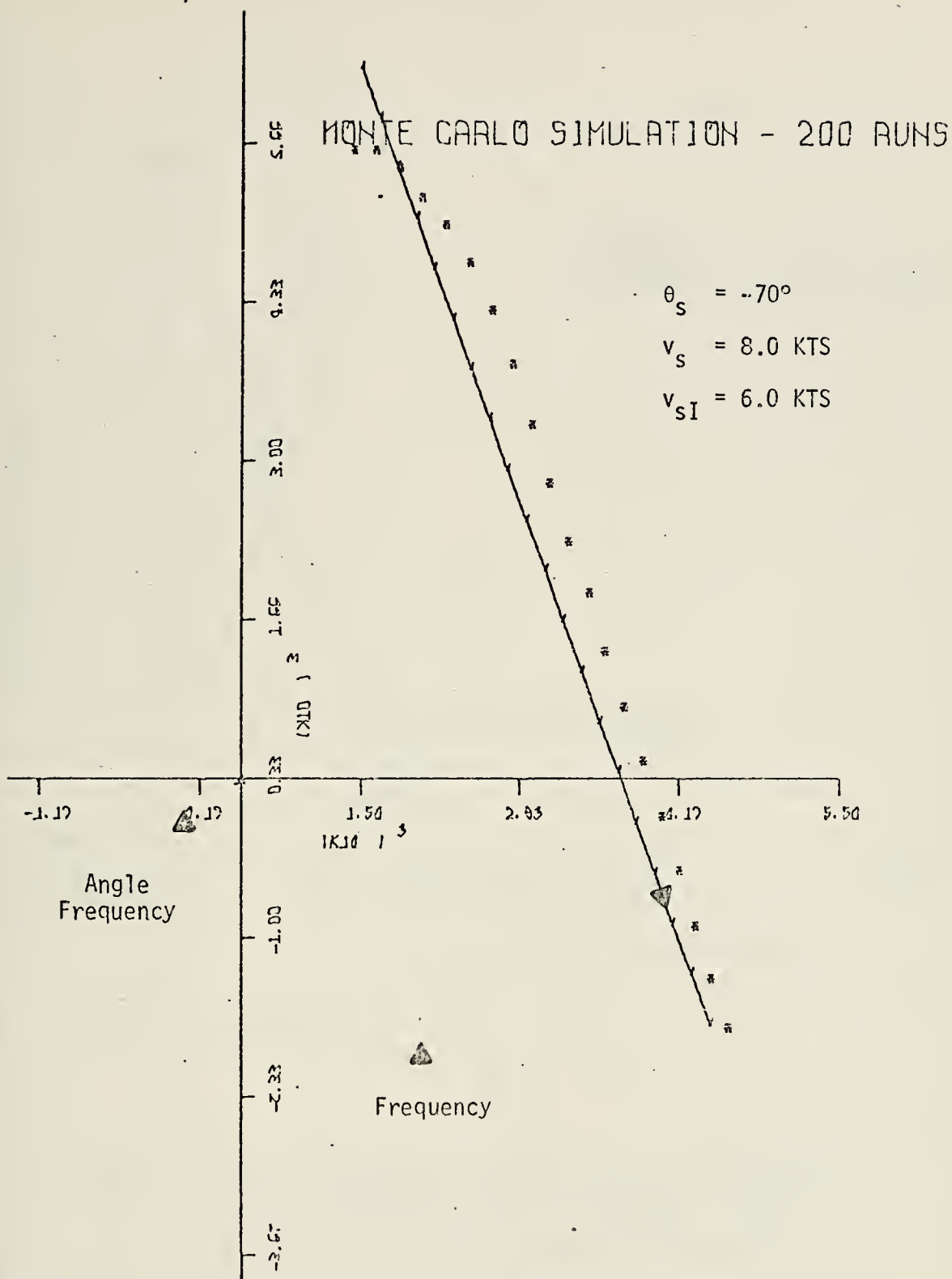


Fig. 5.17 X-Y Filter and Target Track -- Two Sensors;
 Angle - Frequency (0,0); Frequency (2000,-2000)

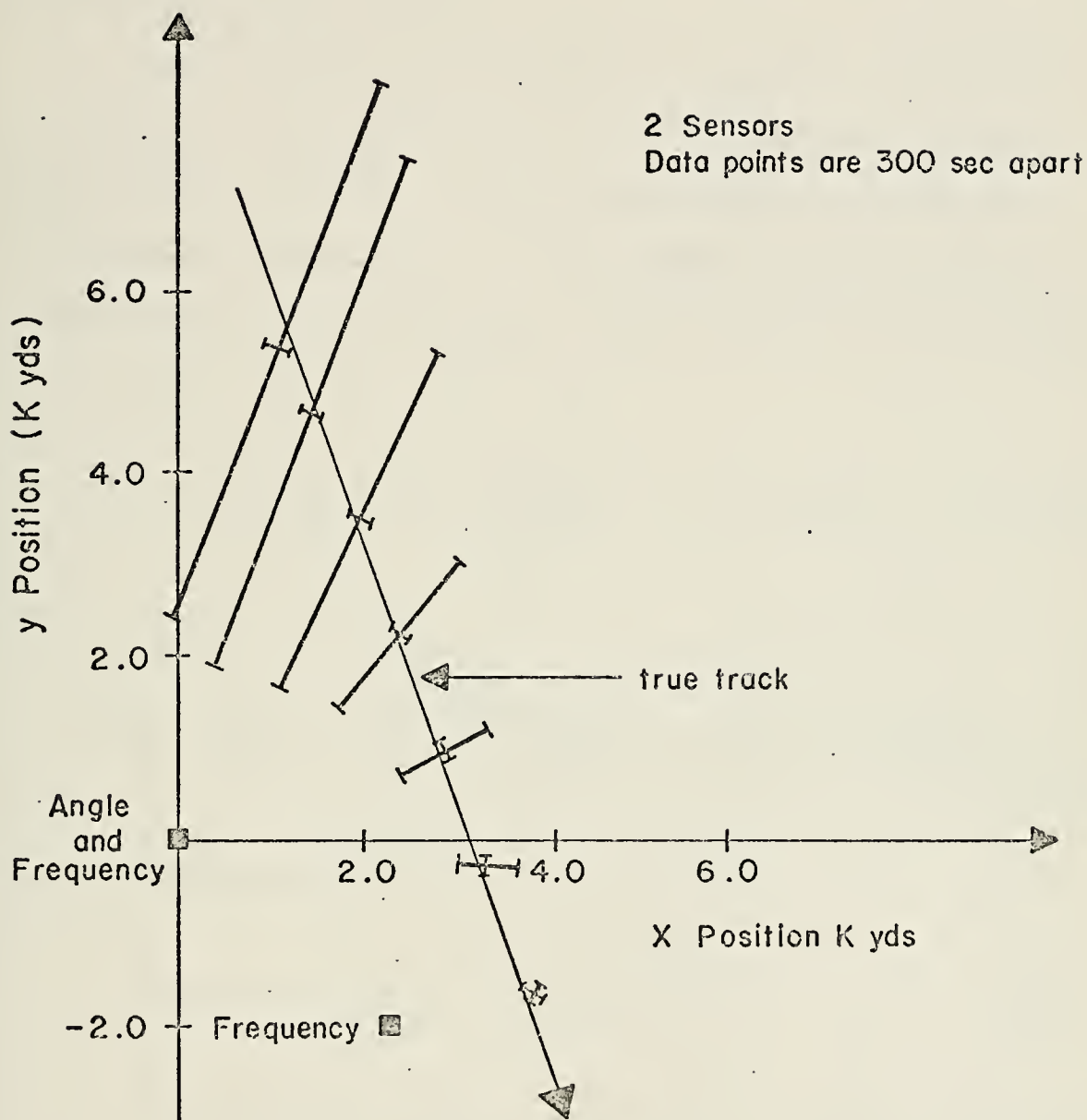


Fig. 5.18 Plot of Major and Minor Axes of Rotated Covariance Ellipses for Position. One Sigma Points are Marked. 2 Sensors

calculated as starting values for each filter (expressed in the appropriate states).

One possible explanation of the poorer performance of the $R_{cpa} - X_{cpa}$ filter lies in the calculation of the position of the target after an observation has occurred. For the X-Y filter, of course, all that is needed are the states X and Y. For the $R_{cpa} - X_{cpa}$ filter the bearing angle to the target must be found from

$$\hat{\theta}(k) = \hat{\theta}_s(k) - \tan^{-1}\left(\frac{A \cdot \hat{R}_{cpa}(k)}{\hat{X}_{cpa}(k)}\right) \quad \text{if } \hat{X}_{cpa}(k) > 0 \quad (5.12a)$$

or

$$\hat{\theta}(k) = \hat{\theta}_s(k) - \tan^{-1}\left(\frac{A \cdot \hat{R}_{cpa}(k)}{\hat{X}_{cpa}(k)}\right) - 180 \quad \text{if } \hat{X}_{cpa}(k) < 0, \quad (5.12b)$$

and the range from

$$\hat{R}(k) = (R_{cpa}(k)^2 + X_{cpa}(k)^2)^{\frac{1}{2}}. \quad (5.13)$$

The position is calculated from the range and bearing. Note that coupling between the position and velocity components has occurred because the heading angle appears in the bearing angle calculation. Thus the position calculation is dependent on three estimated state values instead of the



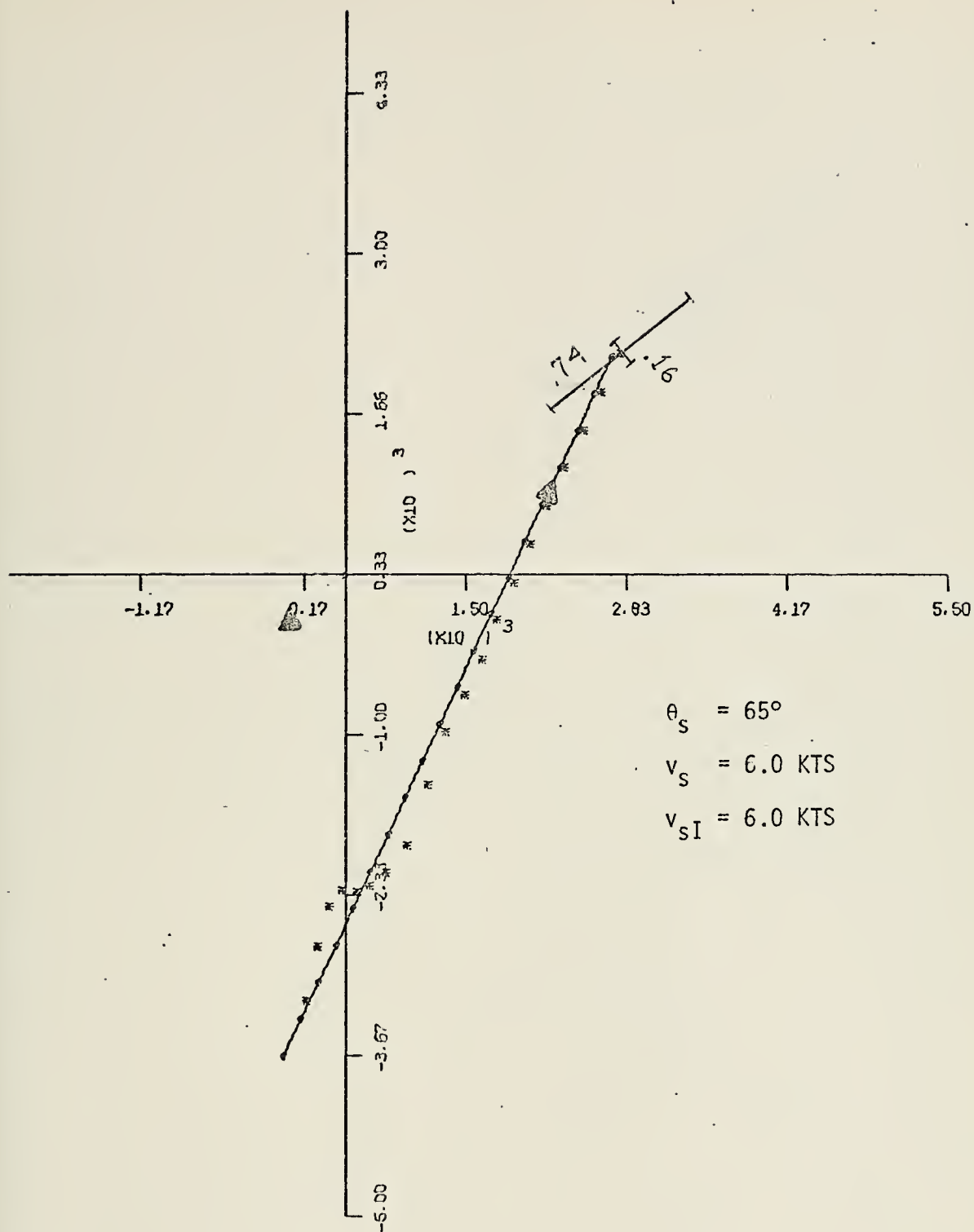


Fig. 5.19 X-Y Filter and Target Track

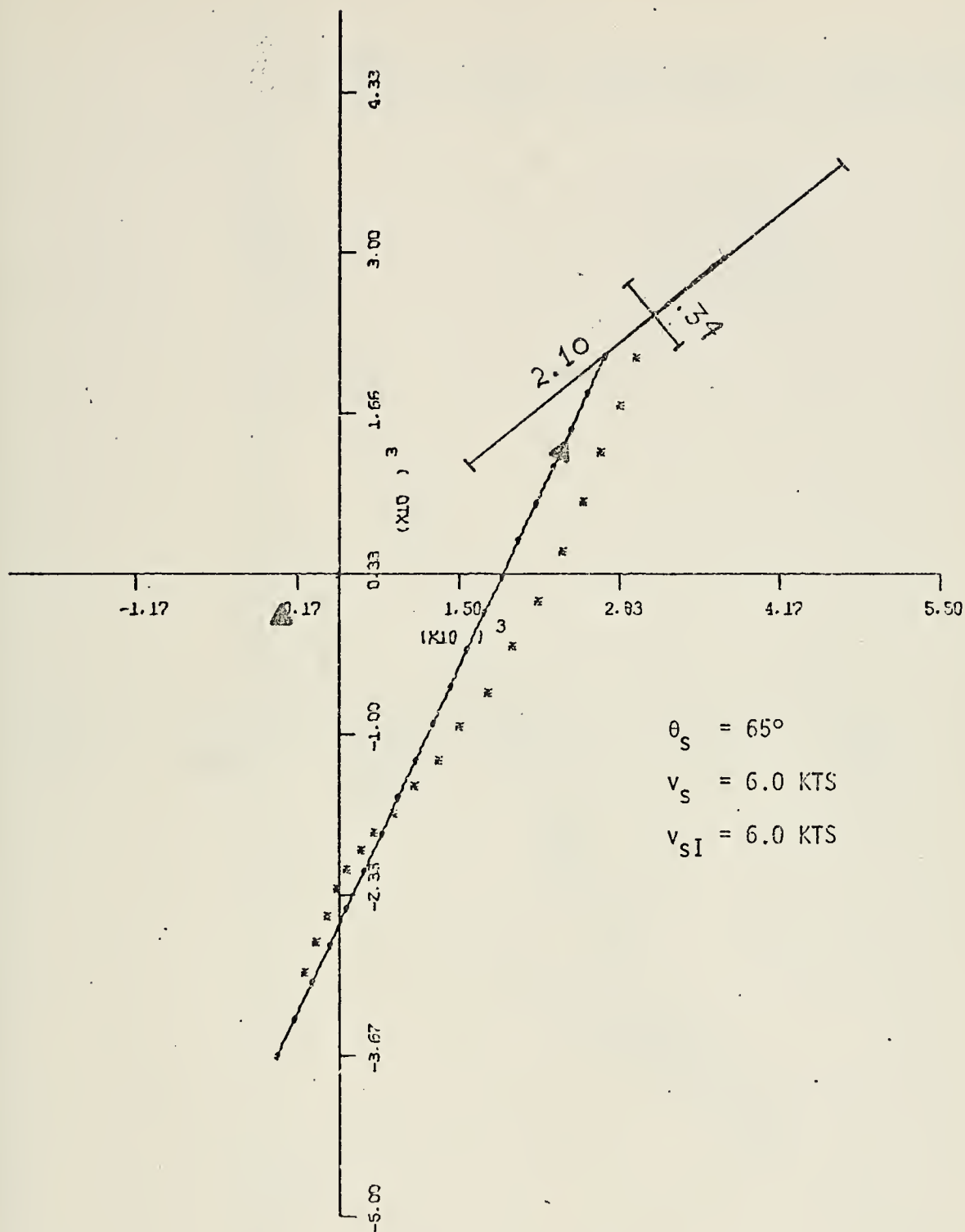


Fig. 5.20 $R_{cpa} - X_{cpa}$ Filter and Target Track

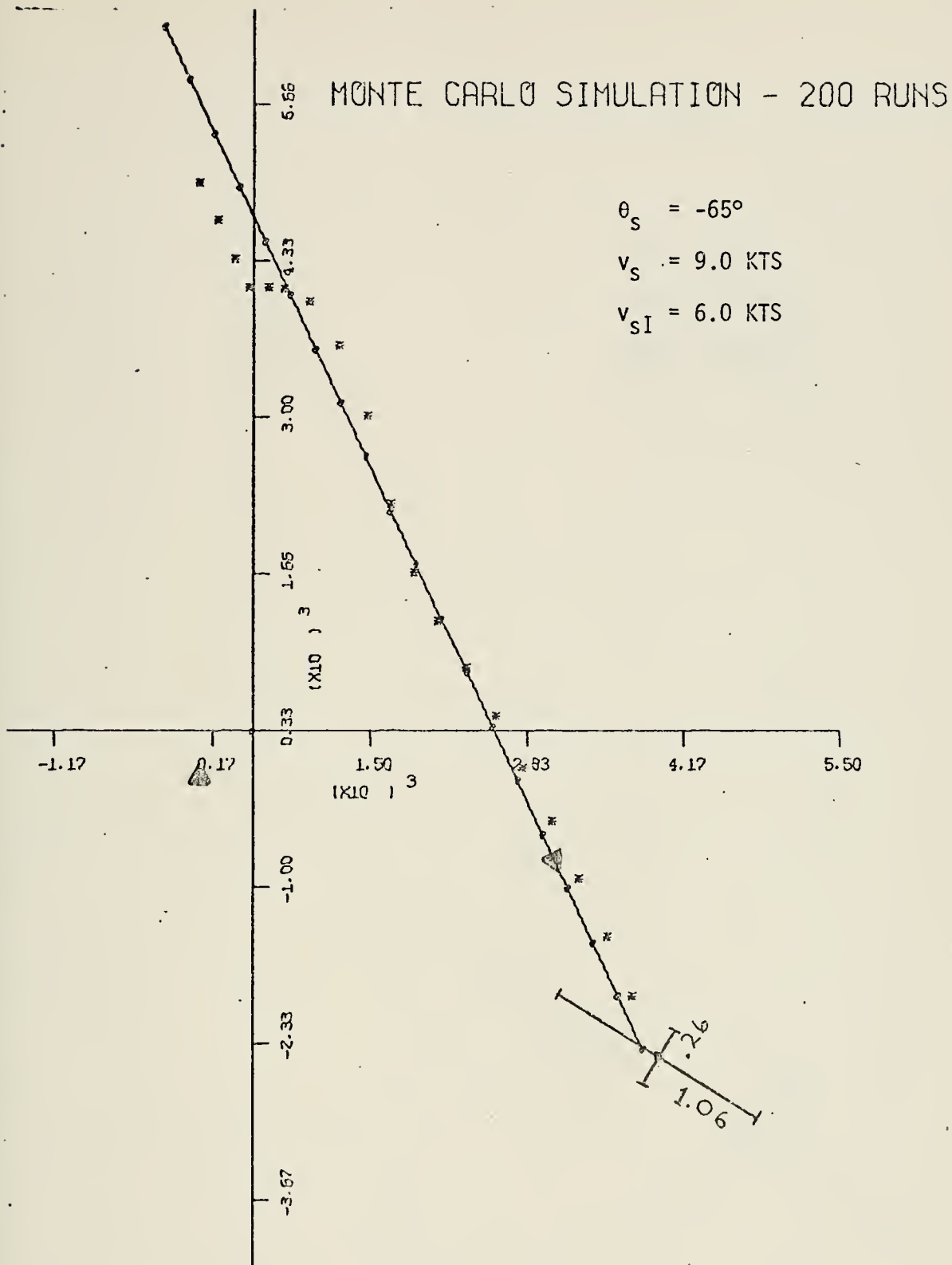


Fig. 5.21 X-Y Filter and Target Track

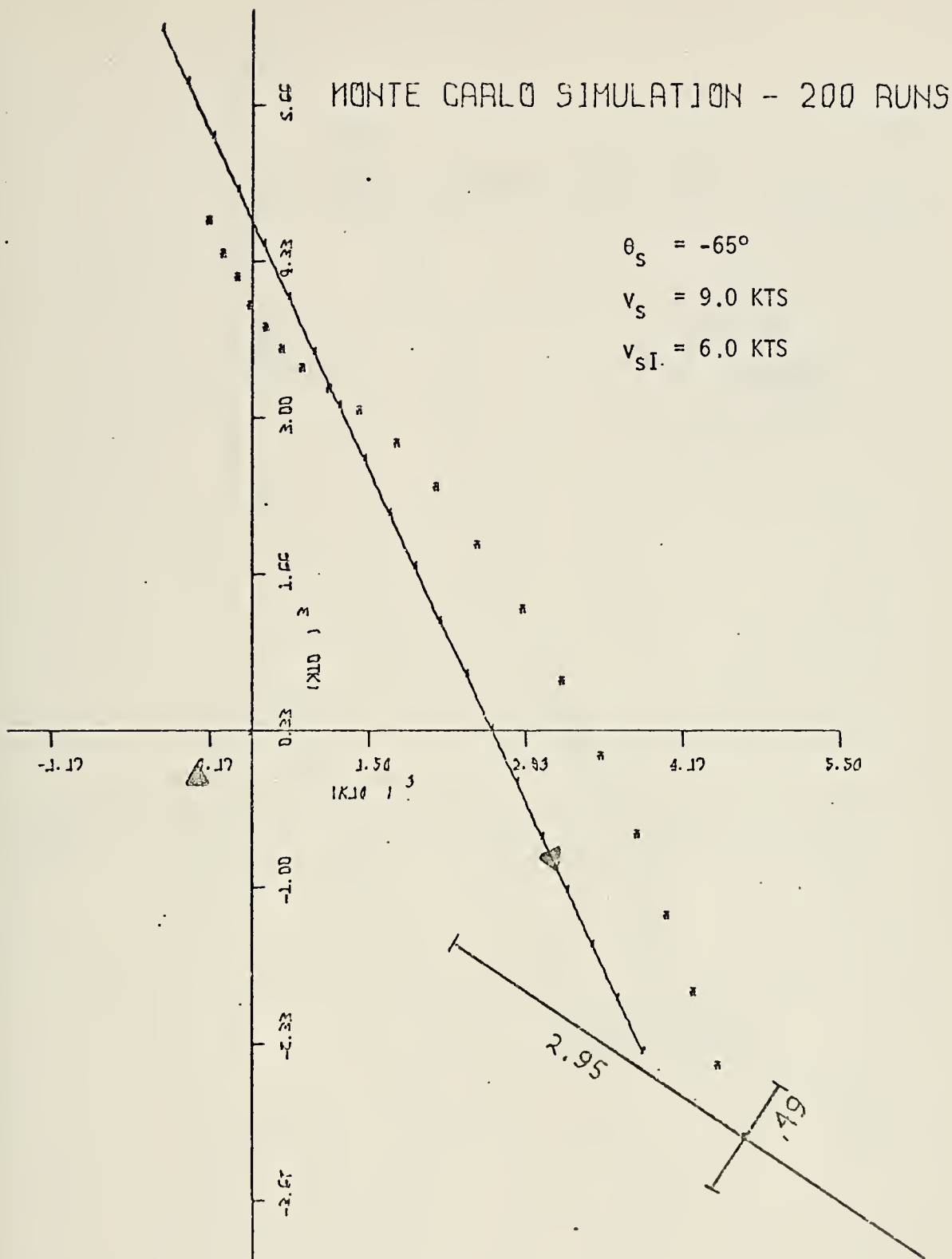


Fig. 5.22 $R_{cpa} - X_{cpa}$ Filter and Target Track

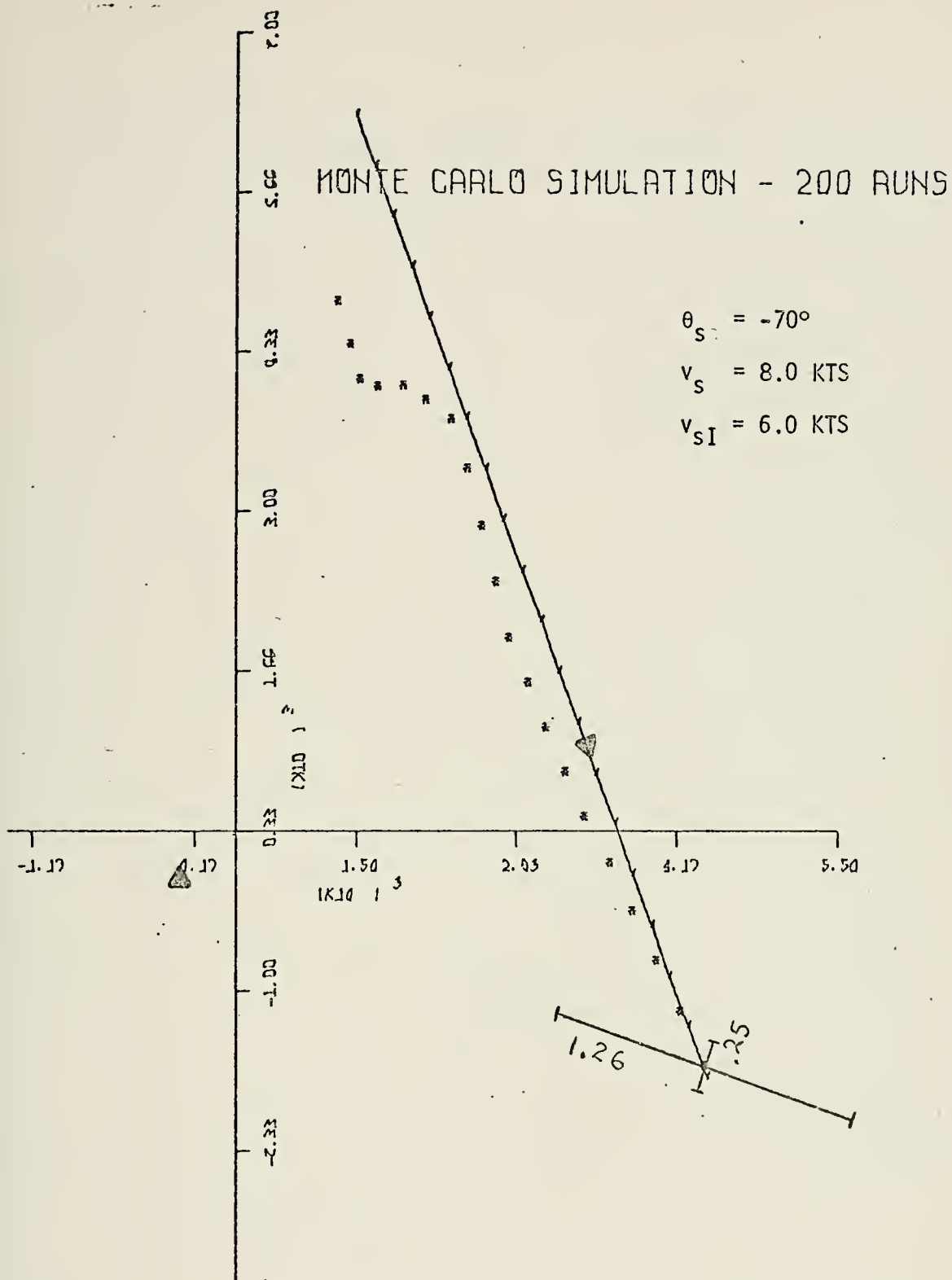


Fig. 5.23 X-Y Filter and Target Track

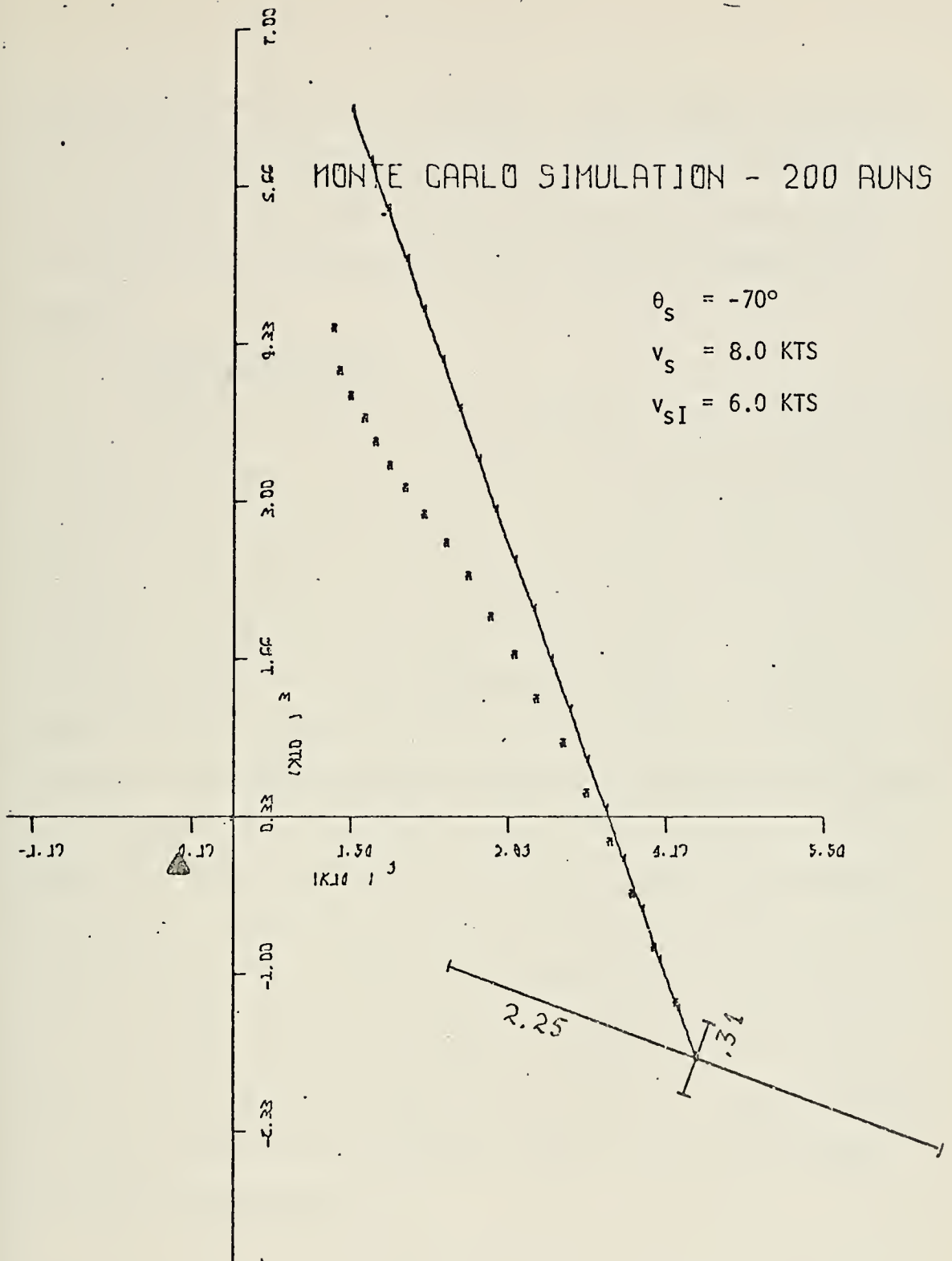


Fig. 5.24 $R_{cpa} - X_{cpa}$ Filter and Target Track

two required for the X-Y filter. A large noise component in the frequency measurement, for example, which results in a significant change in the heading, will thus effect the position by rotating the R_{cpa} , X_{cpa} , R triangle in Fig. 3.2.

5. Filter Residuals

Poirot [34] in his dissertation studied the residuals of an estimation process in order to determine whether biases were present in the procedure. For on-line digital filtering the use of residuals, the difference between the predicted measurement and the actual measurement, might be used for a similar goal. In the simulation program, a counter was established to indicate when the residuals were greater than two standard deviations for the angle and four standard deviations for the frequency noise. The frequency is less sensitive to position errors. These levels were somewhat arbitrary, but for a poor track on a particular run, the counter values were larger when compared to a good track. The effect needs further study but indications are that this could be used to reinitialize the filter and reprocess the data with the new starting values when the residuals show poor performance.

VI. CONCLUSIONS

A. PASSIVE TRACKING AND NONLINEAR FILTERING

The idea of localization and tracking with a single passive sensor has been shown to be feasible. The method of calculating the initial position, heading, and doppler-rest-frequency provides the key that enabled the extended Kalman filter to perform satisfactorily. With only limited a-priori knowledge, the filter was initialized by waiting for a sufficient change in the observations to calculate a good initial condition. The state vector was then brought back to the first observation time to begin filtering. This technique enabled the filter to start with good enough initial conditions so that lock-on and track would result.

The role of the initial covariance matrix was shown to play an important part in the transient response of the filter and resulted in overall better performance as measured by sample means and sample variances in Monte Carlo simulations. That is, the average mean-square error at the end of a given filter run was less when the full initial covariance matrix was used than if only the diagonal terms were used.

The calculation of the covariance of error matrix for this nonlinear problem using a simple Taylor Series expansion was shown to not adequately reflect the actual errors that can be expected due to errors in the measured quantities.

This is due to the linear expansion not being valid over a variation of one standard deviation in these quantities. A method was developed which more accurately estimates the errors by a direct calculation of the changes that occur in the nonlinear function due to a one-standard-deviation change in the input states or measurements. In this way, any physical or geometric boundary conditions can be imposed directly. In an extension of this technique, it is believed that the updating of the covariance matrices used in the linear Kalman filter can be accomplished the same way and the result will be a more accurate weighting of the measurements versus the predicted filter values.

Even with a relatively simple geometric problem, the choice of filter states can play an important role in filter performance. The choice of the best system will be affected by the measurement variables, the coordinate system chosen for the dynamic model, and how any filter excitation is modeled.

An additional sensor measuring frequency-only was found to give significantly lower position error variances than a single angle and frequency sensor. In general it was found that the sensor should view as large a change in the measurement as possible to minimize the effect of measurement noise.

The residuals of the filter, that is, the difference between the actual observation and the predicted observation, were found to be a good qualitative indication of the

transient performance of the filter for a particular run. The values in the calculated filter covariance of error matrix will not show when divergence is occurring. Some criteria using the filter residuals relative to the measurement error can be helpful in this regard. A corrective action might be to reinitialize the filter and start processing the observations over again. The residuals may also be used as an indication of a maneuvering target, in which case the assumed state model is no longer valid.

B. SUGGESTIONS FOR FURTHER STUDY

The most obvious extension of this work would be to test the filtering algorithm on real data. The capability of obtaining the required frequency resolution has been demonstrated [46]. The extreme limits on the angle noise have to be defined so that meaningful measures of the angle noise can be used in the filter.

The method of direct calculation of the covariance matrices for nonlinear problems represents a different approach in applying the linear Kalman filter equations, and this technique should be evaluated against the normal extended Kalman formulation.

Further study of the choice of filter states and their comparison with each other, is another area of interest. The same problem considered here could be formulated in an $R - \theta$ coordinate system for example. In this system both the state and observation equations would be nonlinear however.



An additional area of study might include the effects of a maneuvering target on filter performance, and how the choice of filter states and their coordinate system are involved.

APPENDIX A

Development of Initial Conditions for Single Sensor - The Exact Difference Equations

For the two dimensional constant-speed and constant-heading target, there are five unknowns which completely determine the solution. Expressed in an X-Y coordinate system these are position (x,y) , the velocity (v_x, v_y) and the rest frequency of the emitter, F_0 . The only a priori knowledge available in the general search problem is the approximate target speed and an approximate value for F_0 . The measurements consist of a noisy angle of arrival and a noisy doppler-shifted frequency. Using the change in angle and the change in frequency, an expression for target range and target heading can be found which is not sensitive to knowledge of the rest frequency. The accuracy of the range and heading calculation can be controlled by waiting until the changes in angle and frequency are large with respect to the standard deviations of the noises involved. An expression was derived from recognizing that the rate of change of the doppler-shifted frequency is range dependent. Given range and speed, the heading is then found from the angular velocity expression.

From Fig. A-1, at time t , an angle θ_1 , and frequency f_1 are measured and at time $t+\Delta T$, θ_2 and f_2 are measured. The doppler equations are

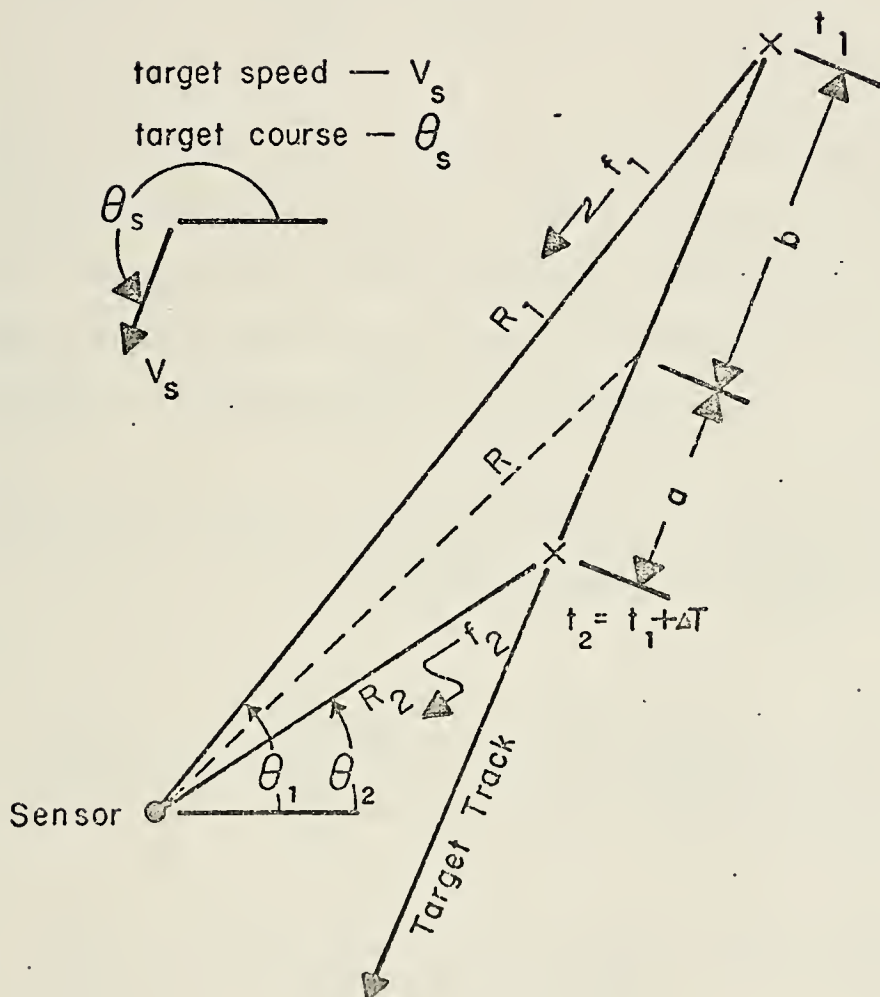


Fig. A-1 Geometry of Sensor and Target.

$$f_1 = F_o \frac{1}{1 + \frac{v_s}{v_p} \cos(\theta_s - \theta_1)} , \quad (A-1)$$

and

$$f_2 = F_o \frac{1}{1 + \frac{v_s}{v_p} \cos(\theta_s - \theta_2)} . \quad (A-2)$$

where F_o is the rest frequency and v_p is the velocity of propagation of the signal. Time differences in the arrival of the signal have been neglected since they are small compared with the time scale of changes in the frequency and angle. Subtracting A-1 from A-2

$$\begin{aligned} \Delta f &\stackrel{\Delta}{=} f_2 - f_1 = F_o \left[\frac{1}{1 + \frac{v_s}{v_p} \cos(\theta_s - \theta_2)} - \frac{1}{1 + \frac{v_s}{v_p} \cos(\theta_s - \theta_1)} \right] \\ &= \frac{F_o v_s}{v_p} \cdot \frac{\cos(\theta_s - \theta_1) - \cos(\theta_s - \theta_2)}{\left[1 + \frac{v_s}{v_p} \cos(\theta_s - \theta_2)\right] \cdot \left[1 + \frac{v_s}{v_p} \cos(\theta_s - \theta_1)\right]} \\ &= \frac{F_o v_s}{v_p} \cdot \frac{-2 \sin \frac{1}{2} [2\theta_s - (\theta_1 + \theta_2)] \cdot \sin \frac{1}{2} (\theta_2 - \theta_1)}{(F_o/f_1) \cdot (F_o/f_2)} . \end{aligned}$$

Let $\Delta\theta = \theta_2 - \theta_1$ and

$$\theta_{AVE} = \frac{\theta_1 + \theta_2 + \theta_3 + \dots + \theta_{KJ}}{KJ} ,$$

where θ_i , $i = 1 - KJ$, are the angle measurements taken at equal time increments between t and $t + \Delta T$. Refer also to Fig. 3.3.

Then

$$\Delta f = - \frac{v_s \cdot f_1 \cdot f_2}{v_p \cdot F_o} \cdot 2 \sin(\theta_s - \theta_{AVE}) \cdot \sin \frac{\Delta \theta}{2} .$$

Solving for $\sin(\theta_s - \theta_{AVE})$ yields

$$\sin(\theta_s - \theta_{AVE}) = - \frac{\Delta f \cdot F_o \cdot v_p}{2 \sin \frac{\Delta \theta}{2} \cdot f_1 \cdot f_2 \cdot v_s} . \quad (A-3)$$

There are two solutions to this equation

$$\theta_s - \theta_{AVE} = \sin^{-1} \left[- \frac{\Delta f \cdot F_o \cdot v_p}{2 \sin \frac{\Delta \theta}{2} \cdot f_1 \cdot f_2 \cdot v_s} \right] ,$$

and

$$180^\circ - (\theta_s - \theta_{AVE}) = \sin^{-1} \left[- \frac{\Delta f \cdot F_o \cdot v_p}{2 \sin \frac{\Delta \theta}{2} \cdot f_1 \cdot f_2 \cdot v_s} \right] .$$

The first solution is the down-doppler or receding target and the second solution is the up-doppler or approaching target. Thus

$$\theta_{SDWN} = \theta_{AVE} + \sin^{-1} \left[- \frac{\Delta f \cdot F_o \cdot v_p}{2 \sin \frac{\Delta \theta}{2} \cdot f_1 \cdot f_2 \cdot v_s} \right] \quad (A-4)$$

and

$$\theta_{sUP} = 180^\circ + \theta_{AVE} - \sin^{-1} \left[- \frac{\Delta f \cdot F_o \cdot v_p}{2 \sin \frac{\Delta \theta}{2} \cdot f_1 \cdot f_2 \cdot v_s} \right] . \quad (A-5)$$

The inverse sine is taken in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. From the approximate knowledge of F_o the up or down-doppler case can usually be determined. For small angles and since

$$\frac{F_o}{f_1 \cdot f_2} \approx \frac{1}{f_2} \quad \text{the expressions simplify to}$$

$$\theta_{sDWN} = \theta_{AVE} + \sin^{-1} \left[- \frac{\Delta f \cdot v_p}{\Delta \theta \cdot f_2 \cdot v_s} \right] \quad (A-6)$$

and

$$\theta_{sUP} = \theta_{AVE} + 180^\circ - \sin^{-1} \left[- \frac{\Delta f \cdot v_p}{\Delta \theta \cdot f_2 \cdot v_s} \right] . \quad (A-7)$$

Using the Law of Sines on the two triangles in Fig. A-1 gives

$$\frac{a}{\sin \frac{\Delta \theta}{2}} = \frac{R}{\sin(\theta_s - \theta_2)} ,$$

and

$$\frac{b}{\sin \frac{\Delta \theta}{2}} = \frac{R}{\sin(\theta_s - \theta_2)} .$$

Now $a+b$ is the distance traveled in time ΔT and thus

$$a+b = v_s \cdot \Delta T = R \sin \frac{\Delta \theta}{2} \cdot \left[\frac{1}{\sin(\theta_s - \theta_2)} + \frac{1}{\sin(\theta_s - \theta_1)} \right]$$

$$= R \sin \frac{\Delta \theta}{2} \cdot \frac{\sin(\theta_s - \theta_1) + \sin(\theta_s - \theta_2)}{\sin(\theta_s - \theta_2) \cdot \sin(\theta_s - \theta_1)}$$

$$= R \sin \frac{\Delta \theta}{2} \cdot \frac{2 \sin(\theta_s - \theta_{AVE}) \cdot \cos \frac{\Delta \theta}{2}}{\frac{1}{2} [\cos \Delta \theta - \cos 2(\theta_s - \theta_{AVE})]}$$

Solving for R and substituting for $\cos 2(\theta_s - \theta_{AVE}) =$

$1 - 2 \sin^2(\theta_s - \theta_{AVE})$, and $\sin \frac{\Delta \theta}{2} \cos \frac{\Delta \theta}{2} = \frac{1}{2} \sin \Delta \theta$, the

result is

$$R = \frac{v_s \cdot \Delta T}{\sin \Delta \theta} \left[\frac{2 \sin^2(\theta_s - \theta_{AVE}) + \cos \Delta \theta - 1}{2 \sin^2(\theta_s - \theta_{AVE})} \right]$$

$$= \frac{v_s \cdot \Delta T \cdot \sin(\theta_s - \theta_{AVE})}{\sin \Delta \theta} \cdot \left[1 + \frac{\cos \Delta \theta - 1}{2 \sin^2(\theta_s - \theta_{AVE})} \right]$$

Substituting for $\sin(\theta_s - \theta_{AVE})$ from equation (A-3)

$$R = - \frac{\Delta f \cdot v_p \cdot F_o \cdot \Delta T}{2 \sin \frac{\Delta \theta}{2} \cdot \sin \Delta \theta \cdot f_1 \cdot f_2} \left[1 + (\cos \Delta \theta - 1) \cdot 2 \cdot \sin^2 \frac{\Delta \theta}{2} \left(\frac{v_s f_1 f_2}{\Delta f F_o v_p} \right)^2 \right]$$

$$= - \frac{\Delta f \cdot v_p \cdot F_o \cdot \Delta T}{2 \sin \frac{\Delta \theta}{2} \cdot \sin \Delta \theta \cdot f_1 \cdot f_2} \left[1 - (1 - \cos \Delta \theta)^2 \cdot \left(\frac{v_s \cdot f_1 \cdot f_2}{\Delta f \cdot F_o \cdot v_p} \right)^2 \right]$$

For typical values of $\Delta\theta$, Δf , v_s , v_p , f_1 , f_2 and F_o , the magnitude of the second term is small compared to one and can be neglected. For example with

$$\Delta\theta = 15^\circ$$

$$\Delta f = .2 \text{ Hz}$$

$$\frac{f_1 \cdot f_2}{F_o} = 500 \text{ Hz}$$

$$v_s = 5.63 \text{ yds/sec}$$

$$(1 - \cos \Delta\theta)^2 \left(\frac{v_s \cdot f_1 \cdot f_2}{\Delta f \cdot F_o \cdot v_p} \right)^2 = .0837$$

Now using the approximations used previously for the heading calculation and neglecting the term above

$$R \approx - \frac{\Delta f \cdot v_p \cdot \Delta T}{(\Delta\theta)^2 f_2}$$

The error variance of the Range and heading calculated above can be determined from the equations in Appendix B and a knowledge of the variance of Δf , $\Delta\theta$, and v_s . Let

$$\text{var}(\Delta f) = 2\sigma_f^2$$

$$\text{var}(\Delta\theta) = 2\sigma_\theta^2$$

$$\text{var}(v_s) = \sigma_{v_s}^2$$

Then from equation (B-1)

$$\begin{aligned}\text{var}(R) &\approx \left(\frac{\partial R}{\partial \Delta\theta}\right)^2 2\sigma_\theta^2 + \left(\frac{\partial R}{\partial \Delta f}\right)^2 2\sigma_f^2 \\ &= R^2 \left[8 \frac{\sigma_\theta^2}{\Delta\theta^2} + 2 \frac{\sigma_f^2}{\Delta f^2} \right]\end{aligned}$$

$$\text{var}(\theta_s) \approx \left(\frac{\partial \theta_s}{\partial \theta_{AVE}}\right)^2 \sigma_{\theta_{AVE}}^2 + \left(\frac{\partial \theta_s}{\partial \Delta\theta}\right)^2 \sigma_{\Delta\theta}^2 + \left(\frac{\partial \theta_s}{\partial \Delta f}\right)^2 \sigma_{\Delta f}^2 + \left(\frac{\partial \theta_s}{\partial v_s}\right)^2 \sigma_{v_s}^2$$

This is shown in Appendix B to be

$$\text{var}(\theta_s) = \frac{\sigma_\theta^2}{KJ} + \frac{R_{cpaI}^2}{X_{cpaI}^2} \left[2 \frac{\sigma_f^2}{\Delta f^2} + 2 \frac{\sigma_\theta^2}{\Delta\theta^2} + \frac{\sigma_{v_s}^2}{v_s^2} \right]$$

$$\begin{aligned}\text{For} \quad \Delta\theta &= 15^\circ & \sigma_\theta^2 &= (5^\circ)^2 \\ \Delta f &= .2 \text{ Hz} & \sigma_f^2 &= (.02 \text{ Hz})^2 \\ v_s &= 5.63 \text{ yds/sec} & \sigma_{v_s}^2 &= (1.68 \text{ yds/sec})^2\end{aligned}$$

$$\text{var}(R) = R^2 \left(\frac{8}{9} + .02 \right) \approx .9R^2 = (3600 \text{ yd})^2$$

$$R = 3830 \text{ yds}$$

$$\begin{aligned}\text{var}(\theta_s) &= .0038 + .246[.02 + .22 + .04] \\ &= (15^\circ)^2\end{aligned}$$

APPENDIX B

Covariance Expressions for Single Buoy Initial Conditions - R_{cpa} - X_{cpa} Filter

The initial conditions for the filter are calculated from the three measurements described in the text: Δf , the frequency difference; $\Delta\theta$, the angle difference; and v_{SI} , the assumed target velocity. It is assumed that Δf and $\Delta\theta$ are independent, normally distributed random variables whose means are unknown and whose variances are obtained from the measurement accuracy. The variance of v_{SI} is based on the a priori knowledge of target speed and it will be assumed to be $(3 \text{ knots})^2$. If each frequency measurement has a variance σ_f^2 , then the difference of two independent measurements has variance $2\sigma_f^2$. Likewise the variance of the angle difference measurement is $2\sigma_\theta^2$. The uncertainty due to the frequency measurement itself will be neglected since it is small compared to other errors, $(\sigma_f^2/f^2 \ll \sigma_{\Delta\theta}^2/\Delta\theta^2)$. The angle θ_{AVE} is the mean of the angle measurements taken before the filter is initialized

$$\theta_{AVE} = \frac{1}{KJ} \sum_{i=1}^{KJ} \theta(t_i) \quad (B-1)$$

such that

$$\theta(t_{KJ}) - \theta(1) > 3 \cdot \sigma_\theta \quad (B-2)$$

The initial filter conditions are derived from the following expressions given in Appendix A.

$$R_{cpaI} = \frac{A \cdot \Delta f^2 \cdot \Delta T \cdot v_p}{\Delta \theta^3 \cdot v_{sI} \cdot f} \quad (B-3)$$

$$X_{cpaI} = \left[\left(\frac{\Delta f \cdot \Delta T \cdot v_p}{\Delta \theta^2 \cdot f_2} \right)^2 - \left(\frac{\Delta f^2 \cdot \Delta T \cdot v_p}{\Delta \theta^3 \cdot v_{sI} \cdot f_2} \right)^2 \right]^{1/2} \quad (B-4)$$

$$v_{sI} = v_{sI} \quad (B-5)$$

$$\theta_{sI} = \theta_{AVE} \pm \sin^{-1} \left(- \frac{\Delta f \cdot v_p}{\Delta \theta \cdot v_{sI} \cdot f_2} \right) - 0 + 180^\circ \quad (B-6)$$

$$F_{oI} = F_{AVE} \left[1 + \frac{v_{sI}}{v_p} \cos(\theta_{sI} - \theta_{AVE}) \right] \quad (B-7)$$

The initial covariance matrix consists of all second order expectations over the probability space of the four random quantities discussed earlier. The effect of other unknowns (v_p , f_2) are assumed small compared with the above.

Papoulis [32] gives an approximate expression for the mean and variance of a smooth function of two random variables. Below an extension is made for n-random variables and an expression for the covariance between two functions of these variables is found. Since Δf , $\Delta \theta$, and v_s are considered independent random variables, and thus their covariances are zero, the expressions can be simplified. Also since θ_{AVE} is the sum of independent random variables

and $\Delta\theta$ is the difference of two of them, θ_{AVE} and $\Delta\theta$ are independent.

Let \underline{x} be a vector of independent random variables and $g(\underline{x})$ and $h(\underline{x})$ be two scalar valued functions of the vector \underline{x} . It is assumed that $g(\underline{x})$ and $h(\underline{x})$ are sufficiently smooth functions in the region of the mean of \underline{x} , \underline{x}_0 , that a Taylor expansion of $g(\underline{x})$ and $h(\underline{x})$ about \underline{x}_0 can be made. It is also assumed that the probability density functions for \underline{x} is symmetrical about the mean and sufficiently compact so that central moments higher than the fourth do not need to be considered. If $g(\underline{x})$ is expanded about its mean, then

$$g(\underline{x}) = g(\underline{x}_0) + \frac{\partial g}{\partial \underline{x}}^T (\underline{x} - \underline{x}_0) + \frac{1}{2} (\underline{x} - \underline{x}_0)^T \left[\frac{\partial^2 g}{\partial x_i \partial x_j} \right] (\underline{x} - \underline{x}_0) + \dots \quad (B-8)$$

with derivatives evaluated at $\underline{x} = \underline{x}_0$. The vector $\frac{\partial g}{\partial \underline{x}}$ is defined by

$$\left(\frac{\partial g}{\partial \underline{x}} \right)^T \triangleq \left[\frac{\partial g_1}{\partial x_1}, \frac{\partial g_2}{\partial x_2}, \dots, \frac{\partial g}{\partial x_n} \right] \quad (B-9)$$

the matrix $\left[\frac{\partial^2 g}{\partial x_i \partial x_j} \right]$ is given by

$$\left[\frac{\partial^2 g}{\partial x_i \partial x_j} \right] \triangleq \begin{bmatrix} \frac{\partial^2 g}{\partial x_1^2} & \frac{\partial g}{\partial x_1 \partial x_2} & \dots & \frac{\partial^2 g}{\partial x_1 \partial x_n} \\ \frac{\partial^2 g}{\partial x_2 \partial x_1} & \frac{\partial g}{\partial x_2^2} & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 g}{\partial x_n \partial x_1} & \dots & \dots & \frac{\partial^2 g}{\partial x_n^2} \end{bmatrix} \quad (B-10)$$

Taking the expectation of the above expression and remembering that the density of \underline{x} is symmetrical and that covariances of the form $E[(x_i - x_{i0})(x_j - x_{j0})] = 0$ because of the independence condition, the following result is obtained.

$$E(g(\underline{x})) = g(\underline{x}_0) + \frac{1}{2} \frac{\partial^2 g}{\partial x_k^2} \sigma_{x_k}^2 \quad (B-11)$$

with

$$\sigma_{x_k}^2 = E[(x_k - x_{k0})^2] \quad (B-12)$$

Substituting $[g(\underline{x})]^2$ for $g(\underline{x})$ above

$$E[g(\underline{x})^2] = g(\underline{x}_0)^2 + \sum_k \left[g \frac{\partial^2 g}{\partial x_k^2} + \left(\frac{\partial g}{\partial x_k} \right)^2 \right] \sigma_{x_k}^2 \quad (B-13)$$

The variance

$$\text{var}[g(\underline{x})] = E[g(\underline{x})^2] - E[g(\underline{x})]^2$$

$$\approx g(\underline{x}_0)^2 + \sum_k \left[g \frac{\partial^2 g}{\partial x_k^2} + \left(\frac{\partial g}{\partial x_k} \right)^2 \right] \sigma_{x_k}^2 -$$

$$g(\underline{x}_0)^2 + g(\underline{x}_0) \sum_k \frac{\partial^2 g}{\partial x_k^2} \sigma_{x_k}^2 + \frac{1}{4} \left[\sum_k \frac{\partial^2 g}{\partial x_k^2} \sigma_{x_k}^2 \right]^2 \quad (B-14)$$

The last term is neglected since it involves fourth order terms. Thus

$$\text{var}[g(\underline{x})] \approx \sum_k \left(\frac{\partial g}{\partial x_k} \right)^2 \sigma_{x_k}^2 = \sum_k \left(\frac{\partial g}{\partial x_k} \sigma_{x_k} \right)^2 \quad (\text{B-15})$$

Similarly the product $g(\underline{x}) \cdot h(\underline{x})$ is expanded about \underline{x}_0 to give

$$E[g(\underline{x})h(\underline{x})] \approx g(\underline{x}_0)h(\underline{x}_0) + \frac{1}{2} E(\underline{x}-\underline{x}_0)^T \left[\frac{\partial g \cdot h}{\partial x_i x_j} \right] (\underline{x}-\underline{x}_0) \quad (\text{B-16})$$

The matrix $\left[\frac{\partial g \cdot h}{\partial x_i x_j} \right]$ has terms of the form

$$g_{ij} = g \frac{\partial^2 h}{\partial x_i x_j} + \frac{\partial g}{\partial x_i} \cdot \frac{\partial h}{\partial x_j} + \frac{\partial h}{\partial x_i} \cdot \frac{\partial g}{\partial x_j} + h \frac{\partial^2 g}{\partial x_i x_j} \quad (\text{B-17})$$

Again using the independence condition

$$E[g(\underline{x})h(\underline{x})] \approx g(\underline{x}_0)h(\underline{x}_0) + \frac{1}{2} \sum_k \left[g \frac{\partial^2 h}{\partial x_k^2} + 2 \frac{\partial g}{\partial x_k} \cdot \frac{\partial h}{\partial x_k} + h \frac{\partial^2 g}{\partial x_k^2} \right] \sigma_{x_k}^2 \quad (\text{B-18})$$

The covariance is

$$\begin{aligned} \text{cov}[g(\underline{x})h(\underline{x})] &= E[g(\underline{x})h(\underline{x})] - E[g(\underline{x})]E[h(\underline{x})] \\ &\approx \left\{ g(\underline{x}_0)h(\underline{x}_0) + \frac{1}{2} \sum_k \left[g \frac{\partial^2 h}{\partial x_k^2} + 2 \frac{\partial g}{\partial x_k} \frac{\partial h}{\partial x_k} + h \frac{\partial^2 g}{\partial x_k^2} \right] \right\} - \\ &\quad \left\{ g(\underline{x}_0)h(\underline{x}_0) + \frac{1}{2} \sum_k \left[g \frac{\partial^2 h}{\partial x_k^2} + h \frac{\partial^2 g}{\partial x_k^2} \right] \sigma_{x_k}^2 + \right. \\ &\quad \left. \frac{1}{4} \left(\sum_k \frac{\partial^2 g}{\partial x_k^2} \sigma_{x_k}^2 \right) \left(\sum_j \frac{\partial^2 h}{\partial x_j^2} \sigma_{x_j}^2 \right) \right\} \quad (\text{B-19}) \end{aligned}$$

Neglecting the fourth order term, the resulting covariance is

$$\begin{aligned} \text{cov}[g(\underline{x})h(\underline{x})] &= \sum_k \left(\frac{\partial g}{\partial x_k} \right) \cdot \left(\frac{\partial h}{\partial x_k} \right) \sigma_{x_k}^2 \\ &= \sum_k \left(\frac{\partial g}{\partial x_k} \sigma_{x_k} \right) \left(\frac{\partial h}{\partial x_k} \sigma_{x_k} \right) \end{aligned} \quad (\text{B-20})$$

The above expressions for variance and covariance were used to establish the initial covariance matrix. Let

$$x_1 = \Delta f \quad \text{with variance } 2 \sigma_f^2 ,$$

$$x_2 = \Delta \theta \quad \text{with variance } 2 \sigma_\theta^2 ,$$

$$x_3 = v_{sI} \quad \text{with estimated accuracy given by a variance term } \sigma_{v_{sI}}^2 ,$$

$$\text{and } x_4 = \theta_1 \quad \text{with variance } \sigma_\theta^2 .$$

Each of the partial derivatives were evaluated at the measured value of Δf and $\Delta \theta$ and the assumed value of v_{sI} . The resulting terms are listed below for the $R_{cpa} - X_{cpa}$ filter:

$$\text{var}[R_{cpaI}] = R_{cpaI}^2 \left(4 \frac{\sigma_{\Delta f}^2}{\Delta f^2} + 9 \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} + \frac{\sigma_{v_{sI}}^2}{v_{sI}^2} \right) \quad (\text{B-21})$$

$$\begin{aligned} \text{var}[X_{\text{cpaI}}] = & (X_{\text{cpaI}})^{-2} [(X_{\text{cpaI}}^2 - R_{\text{cpaI}}^2)^2 \frac{\sigma_{\Delta f}^2}{\Delta f^2} + \\ & (R_{\text{cpaI}}^2 - 2X_{\text{cpaI}}^2)^2 \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} + R_{\text{cpaI}}^4 \frac{\sigma_{v_{\text{SI}}}^2}{v_{\text{SI}}^2}] \end{aligned} \quad (\text{B-22})$$

$$\text{var}(v_{\text{SI}}) = \sigma_{v_{\text{SI}}}^2 \quad (\text{B-23})$$

$$\text{var}(\theta_s) = \frac{\sigma_{\theta}^2}{KJ} + \frac{R_{\text{cpaI}}^2}{X_{\text{cpaI}}^2} \left[\frac{\sigma_{\Delta f}^2}{\Delta f^2} + \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} + \frac{\sigma_{v_{\text{SI}}}^2}{v_{\text{SI}}^2} \right] \quad (\text{B-24})$$

$$\begin{aligned} \text{var}(F_{\text{OI}}) = & \left(\frac{F_{\text{AVE}} \cdot v_{\text{SI}} \cdot R_{\text{cpaI}}^2}{v_p \cdot R_o \cdot X_{\text{cpaI}}} \right)^2 \left[\frac{\sigma_{\Delta f}^2}{\Delta f^2} + \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} + \right. \\ & \left. \left(\frac{X_{\text{cpaI}}^2}{R_{\text{cpaI}}^2} - 1 \right) \frac{\sigma_{v_{\text{SI}}}^2}{v_{\text{SI}}^2} \right] \end{aligned} \quad (\text{B-25})$$

where

$$F_{\text{AVE}} = \frac{1}{KJ} \sum_{i=1}^{KJ} f(i) \quad ,$$

and

$$R_o^2 = R_{\text{cpaI}}^2 + X_{\text{cpaI}}^2 \quad .$$

The covariance terms are

$$\begin{aligned} \text{cov}[R_{\text{cpaI}} \cdot X_{\text{cpaI}}] = & \frac{A \cdot R_{\text{cpaI}}}{X_{\text{cpaI}}} [X_{\text{cpaI}}^2 \left(2 \frac{\sigma_{\Delta f}^2}{\Delta f^2} + 6 \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} \right) - \\ & R_{\text{cpaI}}^2 \left(2 \frac{\sigma_{\Delta f}^2}{\Delta f^2} + 3 \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} + \frac{\sigma_{v_{\text{SI}}}^2}{v_{\text{SI}}^2} \right)] \quad , \end{aligned} \quad (\text{B-26})$$

with A, the angular rotation parameter.

$$\text{cov}(R_{\text{cpaI}} v_{\text{sI}}) = -A \cdot R_{\text{cpaI}} \cdot \frac{\sigma_{v_{\text{sI}}}^2}{v_{\text{sI}}^2} \quad (\text{B-27})$$

$$\text{cov}[R_{\text{cpaI}} \theta_{\text{sI}}] = -\frac{R_{\text{cpaI}}^2}{X_{\text{cpaI}}} \left(2 \frac{\sigma_{\Delta f}^2}{\Delta f^2} + 3 \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} + \frac{\sigma_{v_{\text{sI}}}^2}{v_{\text{sI}}^2} \right) \quad (\text{B-28})$$

$$\begin{aligned} \text{cov}[R_{\text{cpaI}} \theta_{\text{sI}}] = & \frac{F_{\text{AVE}} \cdot v_{\text{sI}} \cdot R_{\text{cpaI}}}{v_{\text{p}} \cdot R_{\text{o}} \cdot X_{\text{cpaI}}} [R_{\text{cpaI}}^2 \left(2 \frac{\sigma_{\Delta f}^2}{\Delta f^2} + 3 \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} \right. \\ & \left. + \frac{\sigma_{v_{\text{sI}}}^2}{v_{\text{sI}}^2} \right) - X_{\text{cpaI}}^2 \frac{\sigma_{v_{\text{sI}}}^2}{v_{\text{sI}}^2}] \quad (\text{B-29}) \end{aligned}$$

$$\text{cov}(X_{\text{cpaI}} v_{\text{sI}}) = \frac{v_{\text{sI}} \cdot R_{\text{cpaI}}^2}{X_{\text{cpaI}}} \cdot \frac{\sigma_{v_{\text{sI}}}^2}{v_{\text{sI}}^2} \quad (\text{B-30})$$

$$\begin{aligned} \text{cov}[X_{\text{cpaI}} \theta_{\text{sI}}] = & \frac{A \cdot R_{\text{cpaI}}}{X_{\text{cpaI}}^2} [R_{\text{cpaI}}^2 \left(\frac{\sigma_{\Delta f}^2}{\Delta f^2} + \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} + \frac{\sigma_{v_{\text{sI}}}^2}{v_{\text{sI}}^2} \right) - \\ & X_{\text{cpaI}}^2 \left(\frac{\sigma_{\Delta f}^2}{\Delta f^2} + 2 \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} \right)] \quad (\text{B-31}) \end{aligned}$$

$$\begin{aligned} \text{cov}[X_{\text{cpaI}} F_{\text{oI}}] = & \frac{F_{\text{AVE}} \cdot R_{\text{cpaI}}^2 \cdot v_{\text{sI}}}{\text{RangeI} \cdot X_{\text{cpaI}}^2 \cdot v_{\text{p}}} \cdot [X_{\text{cpaI}}^2 \left(\frac{\sigma_{\Delta f}^2}{\Delta f^2} + 2 \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} + \right. \\ & \left. \frac{\sigma_{v_{\text{sI}}}^2}{v_{\text{sI}}^2} \right) - R_{\text{cpaI}}^2 \left(\frac{\sigma_{\Delta f}^2}{\Delta f^2} + \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} + \frac{\sigma_{v_{\text{sI}}}^2}{v_{\text{sI}}^2} \right)] \quad (\text{B-32}) \end{aligned}$$

$$\text{cov}[v_{sI} \theta_{sI}] = \frac{v_{sI} \cdot A \cdot R_{cpaI}}{X_{cpaI}} \cdot \frac{\sigma_{v_{sI}}^2}{v_{sI}^2} \quad (\text{B-33})$$

$$\text{cov}[v_{sI} F_{oI}] = \frac{v_{sI}^2 \cdot F_{AVE}}{v_p \cdot R_o \cdot X_{cpaI}} (X_{cpaI}^2 - R_{cpaI}^2) \frac{\sigma_{v_{sI}}^2}{v_{sI}^2} \quad (\text{B-34})$$

$$\begin{aligned} \text{cov}[\theta_{sI} F_{oI}] = & \frac{A \cdot R_{cpaI} \cdot F_{AVE} \cdot v_{sI}}{v_p \cdot R_o \cdot X_{cpaI}^2} [X_{cpaI}^2 \cdot \frac{\sigma_{v_{sI}}^2}{v_{sI}^2} - \\ & R_{cpaI}^2 (\frac{\sigma_{\Delta f}^2}{\Delta f^2} + \frac{\sigma_{\Delta \theta}^2}{\Delta \theta^2} + \frac{\sigma_{v_{sI}}^2}{v_{sI}^2})] \quad (\text{B-35}) \end{aligned}$$

APPENDIX C

The Probabilities Associated with Measurement Errors for Different Values of Angle Differences

Assume that the angle measurements are independent samples consisting of the true angle plus additive white Gaussian noise. That is

$$\theta_i = \theta_{i0} + \omega_i$$

The probability density function of the ω_i 's is zero-mean with variance σ_θ^2 . Thus each θ_i is normally distributed with mean θ_{i0} and variance σ_θ^2 . The probability distribution of the difference between θ_i and θ_j , $\Delta\theta$, is therefore normally distributed with mean $\theta_{i0} - \theta_{j0}$ and variance $2\sigma_\theta^2$. The measurement error

$$\theta_\epsilon = (\theta_i - \theta_j) - (\theta_{i0} - \theta_{j0})$$

is, as a result, normally distributed with zero mean and variance $2\sigma_\theta^2$. From the probability tables

$$P_r\{|\theta_\epsilon| < a \cdot \sigma_\theta\} = P_a$$

with P_a the probability associated with a normal random variable being within standard deviations of the mean. Substituting the angle variance gives

$$P_r\{|\theta_\epsilon| < a \cdot \sqrt{2} \sigma_\theta\} = P_a$$

If the angle difference $\Delta\theta$ is greater than 3 , then

$$P_r\{|\theta_\epsilon| < a \sqrt{2} \frac{\Delta\theta}{3}\} > P_a .$$

This means that for example

$$P_r\{|\theta_\epsilon| < .5 \Delta\theta\} = P_a$$

where

$$\frac{\sqrt{2}}{3} a = .5$$

$$\text{or} \quad a = 1.06 .$$

The probability that a normal random variable is within 1.06 standard deviations of its mean is 0.71. Thus for $\Delta\theta > 3\sigma_\theta$ the probability that $\theta_\epsilon/\Delta\theta < .5$ is greater than 0.71.

Figure C1 shows a general graph of

$$P_r\{|\theta_\epsilon| \leq \frac{a \sqrt{2}}{b} \Delta\theta\} = P_a$$

$$\text{with } \Delta\theta = b\sigma_\theta , \quad \text{and} \quad a' = \frac{a \sqrt{2}}{b}$$

as a parameter.

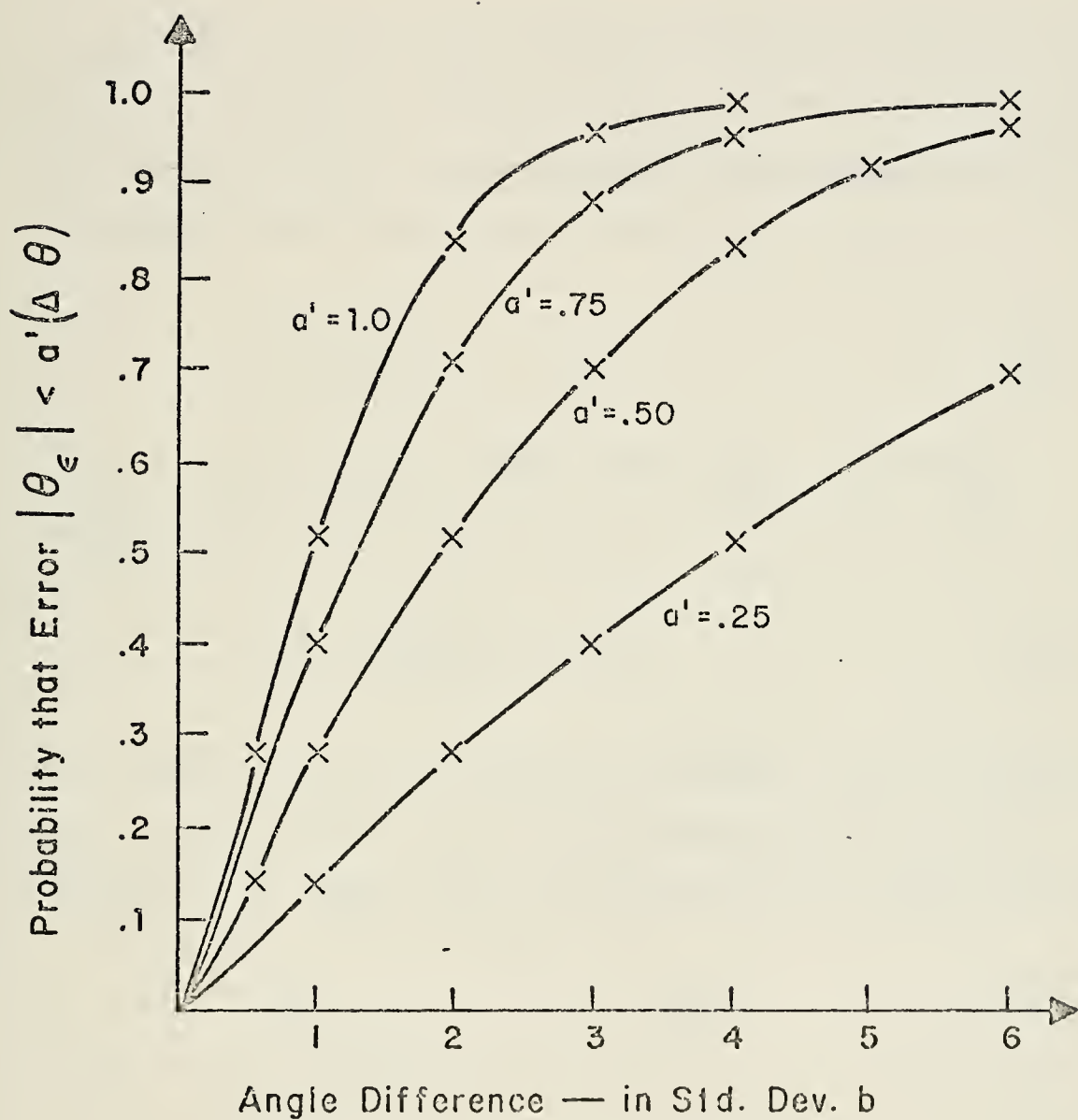


Fig. C-1. Probability of Angle Difference Error for Different Values of Angle Difference.

APPENDIX D

The Confidence Interval for Statistics Calculated using 200 Monte Carlo Runs

Bucy [11] gives an approximate method for estimating the accuracy of computed statistics based on the assumption that the samples are from a normally distributed random variable. The sample mean is given by equation (5.1) and is

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i, \quad (D-1)$$

where N is the number of Monte Carlo runs. The sample variance is

$$S_n^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2. \quad (D-2)$$

Let μ and σ^2 be the true mean and variance of the distribution that the samples are from, and the quantities that are of interest. The sample mean and variance are unbiased, that is

$$E(\bar{x}) = \mu \quad \text{and} \quad (D-3)$$

$$E(S_n^2) = \sigma^2. \quad (D-4)$$

The variance of the mean estimate is given by

$$\text{var}(\bar{x}) = \sigma_{\bar{x}}^2 = E(\bar{x} - \mu)^2 = \frac{E(x_1^2) - \mu^2}{N} = \frac{\sigma^2}{N} \quad (D-5)$$

The variance of the variance estimate is given by

$$\text{var}(S_n^2) = E[S_n^2 - \sigma^2] = \frac{E(x_i^4) - (\sigma^2)^2}{N} \quad (\text{D-6})$$

If the distribution of the x_i 's are approximately normal, then

$$E(x_i^4) \approx 3(\sigma^2)^2, \quad (\text{D-7})$$

and thus

$$\text{var}(S_n^2) = \sigma_{S_n^2}^2 = 2 \frac{(\sigma^2)^2}{N}. \quad (\text{D-8})$$

From the variances for \bar{x} and S_n^2 and probability tables, the confidence region for \bar{x} and S_n^2 can be found. Let N be 200 Monte Carlo runs. Then

$$P_r\{|\bar{x} - \mu| < .1 S_n\} = P_r\{|\bar{x} - \mu| < a \sigma_{\bar{x}}\} \quad (\text{D-9})$$

where $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} \approx \frac{S_n}{\sqrt{N}}$, and a is the number of standard deviations in $.1 S_n$. That is

$$a = \frac{.1 S_n}{S_n / \sqrt{N}} = .1 \sqrt{N}$$

or

$$a = .1 \sqrt{200} = 1.414.$$

The probability that a normal random variable is within 1.414σ of its mean is .84, which means

$$P_r\{|\bar{x} - \mu| < .1 S_n\} = .84 \quad . \quad (D-10)$$

For the variance

$$P_r\{|S_n^2 - \sigma^2| < 2 \sigma_{S_n^2}\} = .95 \quad . \quad (D-11)$$

From equation (D-8)

$$2 \sigma_{S_n^2}^2 = 2 \sqrt{\frac{2}{N}} \sigma^2 \quad .$$

Substituting this into equation (D-11) and writing out the absolute value inequality gives

$$P_r\left\{-\frac{2\sqrt{2}}{\sqrt{N}} \sigma^2 < S_n^2 - \sigma^2 < \frac{2\sqrt{2}}{\sqrt{N}} \sigma^2\right\} = .95 \quad ,$$

or

$$P_r\left\{\frac{S_n^2}{1 + 2\sqrt{\frac{2}{N}}} < \sigma^2 < \frac{S_n^2}{1 - 2\sqrt{\frac{2}{N}}}\right\} = .95 \quad .$$

With $N = 200$ this gives

$$P_r\{.774 S_n^2 < \sigma^2 < 1.39 S_n^2\} = .95 \quad ,$$

or

$$P_r\{.88 S_n < \sigma < 1.18\} = .95 \quad .$$

LIST OF SYSTEM LIBRARY SUBROUTINES USED

GMPRO	Matrix Product
ATAN	Arc Tangent
MINV	Matrix Inverse
ARSIN	Arc Sine
GAUSS	Normal Random Number Generator
PLOTP	Printer Plotting Package
ARCOS	Arc Cosine
SIN, COS, TAN	Trig Functions
GMTRA	Matrix Transpose
SQRT	Square Root

LIST OF PROGRAM VARIABLES COMMON TO BOTH FILTERS

R(I,J)	exact range at time T(J) from sensor I to target
FD(I,J), FDDT(I,J)	exact doppler shifted frequency and rate of change of frequency at time T(J) at sensor I
BR(J)	exact bearing angle to target at time T(J) from sensor I
VR(I,J), VRD T(I,J)	radial velocity and time rate of change of radial velocity at time T(J) at sensor I
XS(J), YS(J)	exact X,Y position of target at time T(J)
X(I), Y(I)	X,Y position of sensor I
T(J)	time of observation
XS, YS	initial X,Y position of target
VSS	exact target velocity in knots
VS	VSS in yds/sec
THTA	exact target heading in degrees
THTAS	THTA in radians
OVX	exact target X velocity
OVY	exact target Y velocity
FREQ	exact rest frequency
VMED, VP	velocity of propagation
SA, AMA	standard deviation and mean for normal angle noise generator
SF, AMF	standard deviation and mean for normal frequency generator
IXA, IXF	starting numbers for noise generator
AVEVA, VARVA	computed mean and variance of random angle noise

NSPOT	number of different tracks to be run
JJQQ	number of runs in Monte Carlo simulation
NUM	number of time points
TINT	time interval between data points
M	number of observation variables
Z(J,N)	noisy observations at time T(J), Index N determines whether it is an angle or frequency and will depend on number and type of sensors
AVSK	initial guess at target velocity in knots
AVS	AVSK in yds/sec
FO	initial guess at rest frequency - used only to determine whether target is up or down doppler
KJ	counter which indicates how many time cuts were needed to satisfy angle difference criterion
ADELTH	angle difference which satisfies criterion, i.e., $ADELTH > 3SA$
ADELT	time difference corresponding to angle difference
ATHALF	time at which initial filter states are found
ADELf	frequency difference corresponding to angle difference
ATHETH	average of all angle measurements during initial condition calculation
FAVE	average of frequency measurements during initial condition calculation
ASINE	Arc sine of angle difference between the the target heading and target bearing
ARHALF	calculated initial target range
FF	one standard deviation of ADELf, equals 2 SF

AA	one standard deviation of ADELTH, equals 2 SA
VV	guess at accuracy of AVSK, equals 3 KTS
AVAV	one standard deviation of ATHETH, equals SA/ KJ
HEAD (ADELF, ADELTH, AVS, ATHETH)	function subprogram to calculate target heading from initial measurements
HUF	target heading when ADELTH is increased by FF
HDF	target heading when ADELTH is decreased by FF
HVA	target heading when ADELTH is increased by AA
HDA	target heading when ADELTH is decreased by AA
HUV	target heading when AVS is increased by VV
HDV	target heading when AVS is decreased by VV
NO	= 0 for full initial covariance matrix, = 1 for only diagonal terms
COVK	initial state covariance matrix, also updated state covariance matrix
COV	predicted state covariance matrix
PHI	state transition matrix
PHITR	PHI transposed
SIGTH	heading angle excitation in $(\text{rad/K sec})^2$
SIGVS	target speed excitation in $((\text{yds/sec})/\text{sec})^2$
SIGFO	target rest frequency excitation $(H_z/\text{Ksec})^2$
F(N)	predicted doppler shifted frequency at sensor N
QEXIT	state excitation matrix
HOB	linearized observation matrix
HOBTR	HOB transposed
GAIN	Kalman Filter gain matrix
GAINTR	GAIN transposed
ZNOIS	observation noise covariance matrix

XCUR	state vector before observation
XPRED	predicted state vector (= $\text{PHI} \cdot \text{XCUR}$)
XFIL	filtered state vector after observation
IRE	residual error counter
XMC(I,J)	Monte Carlo mean of state I for time T(J)
XVAR(I,J)	Monte Carlo variance of state I for time T(J) (See RCPA - XCPA filter)
XTAR(J)	filtered X position of target at time T(J)
YTAR(J)	filtered Y position of target at time T(J)
XINT(Z),YINT(Z)	average of X and Y initial positions for Monte Carlo runs
RFIL(J),THFIL(J)	calculated target range and bearing from filtered states at time T(J)

LIST OF PROGRAM VARIABLES FOR X-Y FILTER

X(1)	X position
X(2)	X velocity
X(3)	Y position
X(4)	Y velocity
X(5)	rest frequency
XUF	X refers to which state, U refers to up or increase, and F refers to which variable is changed. F is frequency term ADELFF. Thus XUF is the calculated X initial position due to an increase in ADELFF by FF
XDF	calculated X initial position due to a decrease in ADELFF by FF
SGXF	average of the change due to increasing ADELFF and the change due to decreasing ADELFF by AA
SGVXA	average of the change due to increasing ADELTH and the change due to decreasing ADELTH by AA
The other terms follow the same pattern	
TT	time interval between measurements
SNEWXV, SNEWXS, SNEWYV, SNEWYS	Monte Carlo variances and standard deviations of X and Y positions in rotated coordinate system
HDING	calculated initial target heading
H1	predicted target bearing from sensor 1 in radians
RESA	angle residual for sensor 1 in radians
RESFK	frequency residual for sensor K in H_z

RERROR(J)	true range error for time T(J) i.e. distance from filtered position to true position
VSFIL(J)	computed target speed from filtered X and Y velocities at time T(J)
HDFIL(J)	computed target heading from filtered X and Y velocities at time T(J)
RTEMP	computed target range from filtered X and Y positions
XYVAR(I,J)	Monte Carlo covariance of X and Y filter positions at time T(J)
XYVAR(Z,J)	Monte Carlo covariance of X and Y filter velocities at time T(J)
XYA(J)	angle of Monte Carlo probability ellipse for position variances
VXVYA(J)	angle of Monte Carlo probability ellipse for velocity variances

LIST OF PROGRAM VARIABLES FOR R_{cpa} , X_{cpa} FILTER

X(1)	range to target at CPA from sensor 1
X(2)	range of target to CPA
X(3)	target speed
X(4)	target heading
X(5)	rest frequency
REX	exact value of Range to CPA from sensor 1
XEX	exact value of Range to CPA from target
NDIFA	number of angle and frequency sensors
NLOFA	number of frequency sensors
NBU	total number of sensors used
W	angular rotation parameter around sensor 1 (= +1 counterclockwise, = -1 clockwise)
RCPAI	initial filter state X(1)
XCPAI	initial filter state X(2)

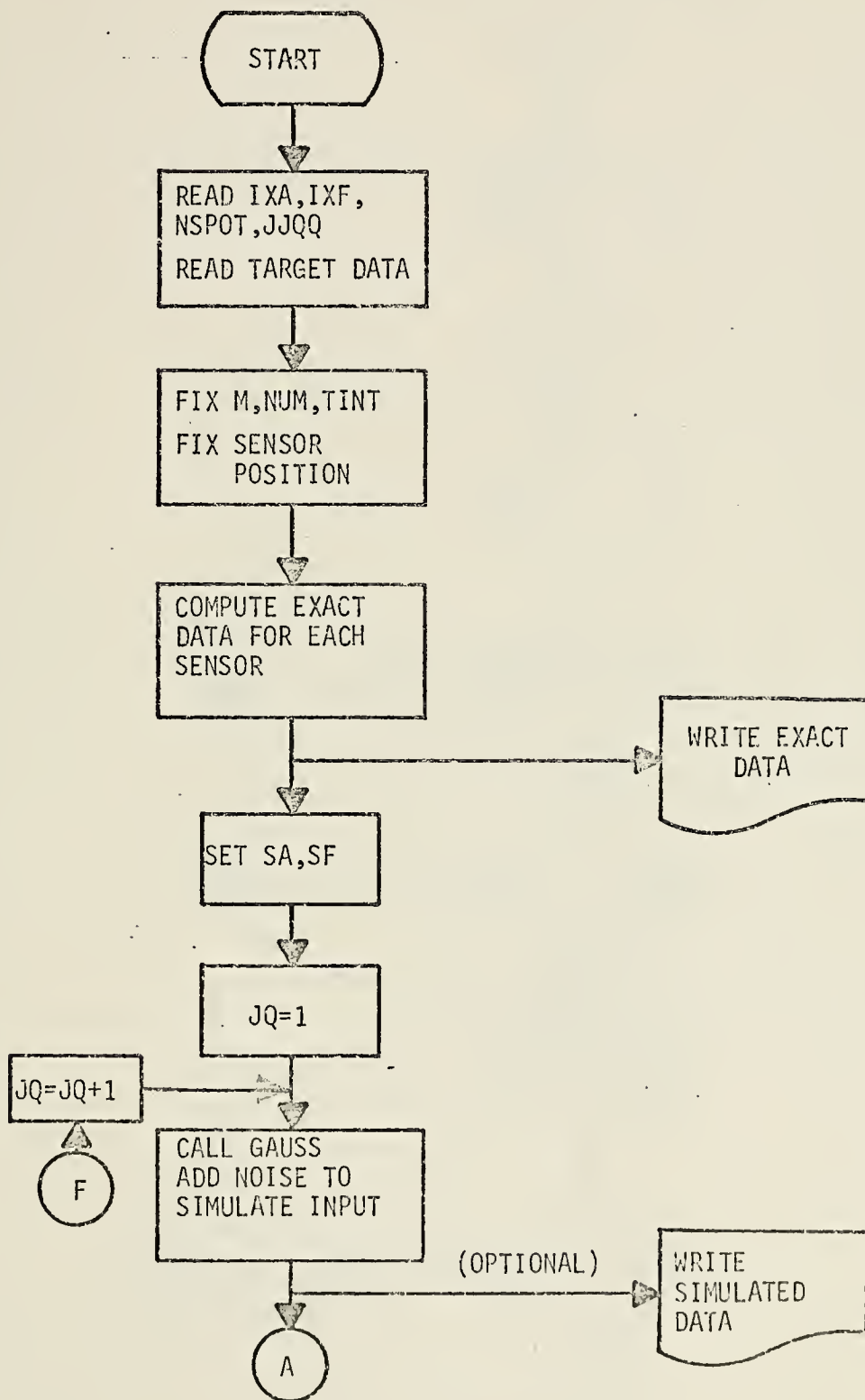
Initial covariance terms are defined in a similar manner as the X-Y filter. Thus RVV is the initial R_{CPA} due to an increase in Δv_s by VV .

DELTAT	time interval between measurements
D(I)	distance from sensor 1 to sensor I
TH(I)	bearing from sensor 1 to sensor I
RCPA(I)	predicted range at CPA from sensor I
XCPA(I)	predicted range to CPA of target for sensor I
RANGE(I)	predicted range from target to sensor I
TH TAX(I)	predicted bearing of target from sensor I
RES 1	angle residual for sensor 1

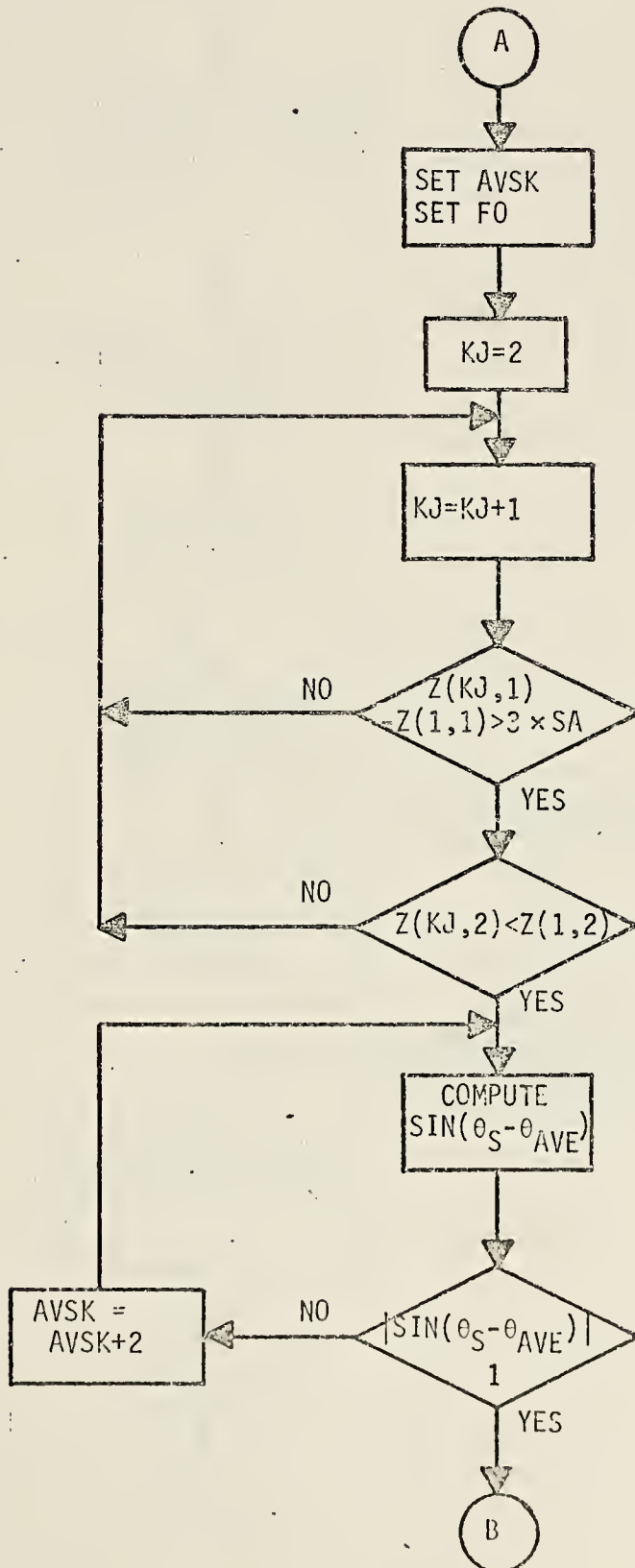
RES 2	frequency residual for sensor 1
RES 3, RES 4	angle and frequency residual depending on number and type of sensors
AVRESA	statistical mean of RES1 for a single run
STDRAZ	statistical standard deviation of RES1 for a single run
AVRF1	statistical mean of RES2 for a single run
STFIZ	statistical variance of RES2 for a single run
ICOUNT	number of times filter has been reinitialized due to residual error counter IRE
IEND	maximum number of times a reinitialization will be tried
XMC(I,J), XMC(Z,J)	Monte Carlo mean of target X and Y position at time T(J)
XVAR(I,J), XVAR(Z,J)	Monte Carlo variance of target X and Y position at time T(J)
XYVAR(I,J)	Monte Carlo covariance of X and Y position at time T(J)
XYVAR(Z,J)	Monte Carlo covariance of target speed and heading at time T(J)

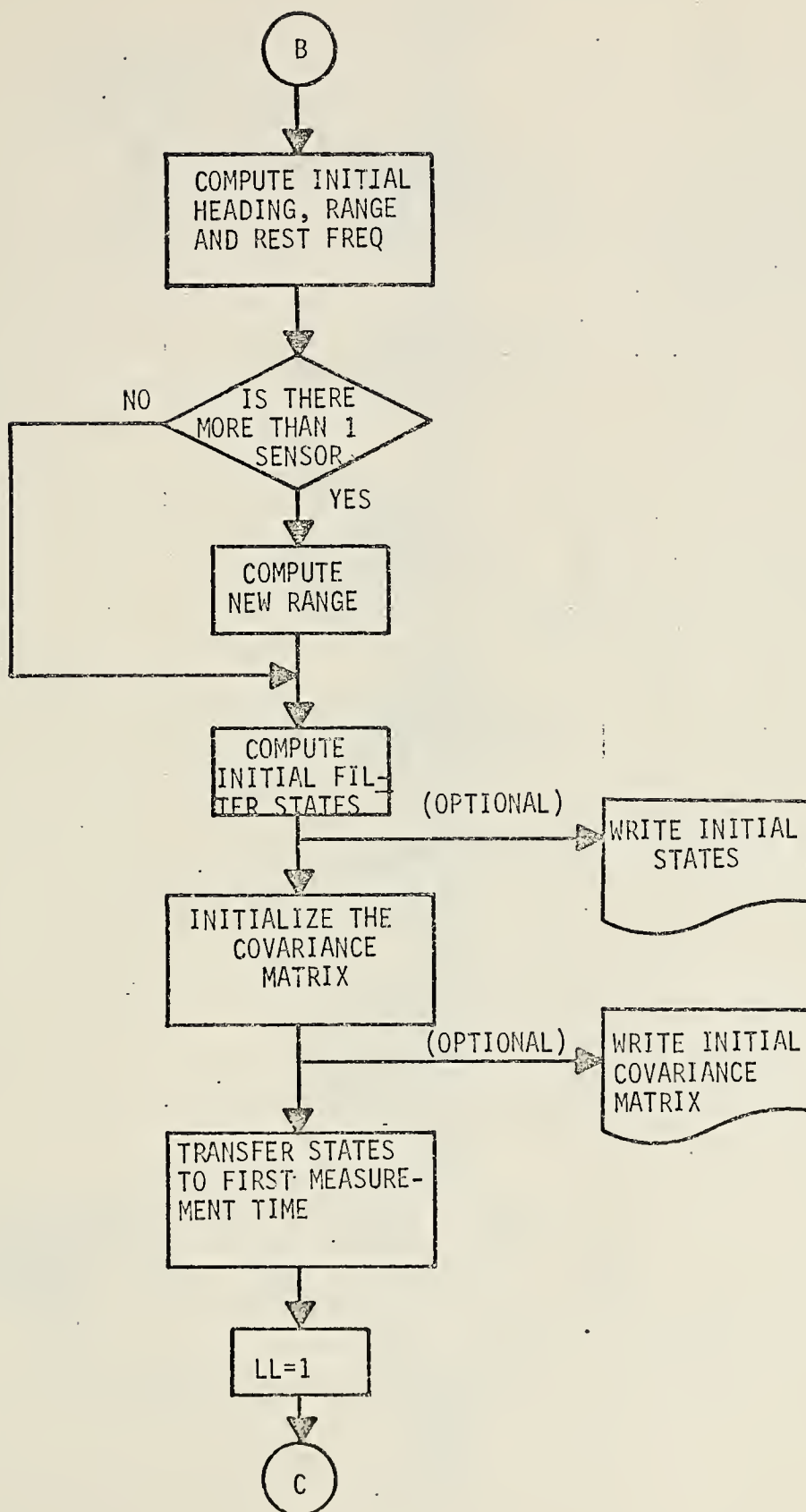


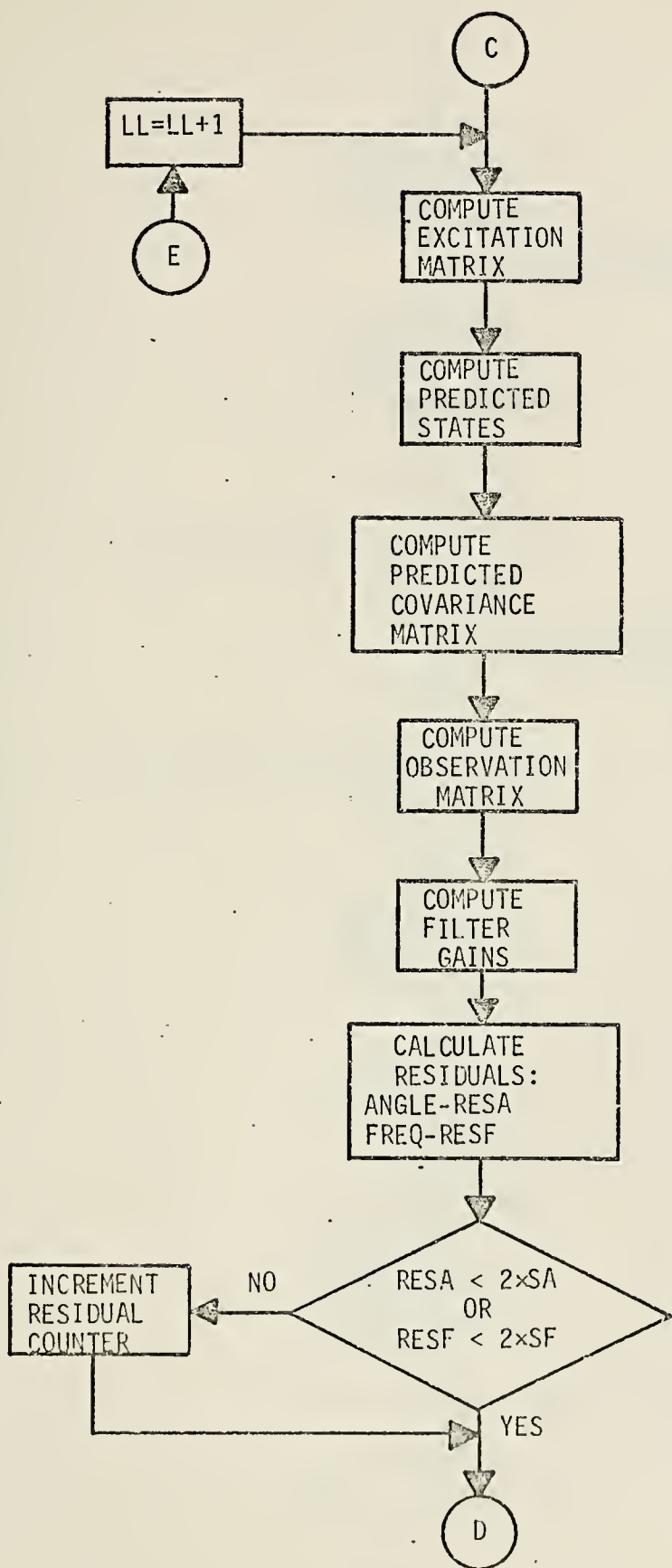
Flow Chart for X-Y Filter Simulation Program

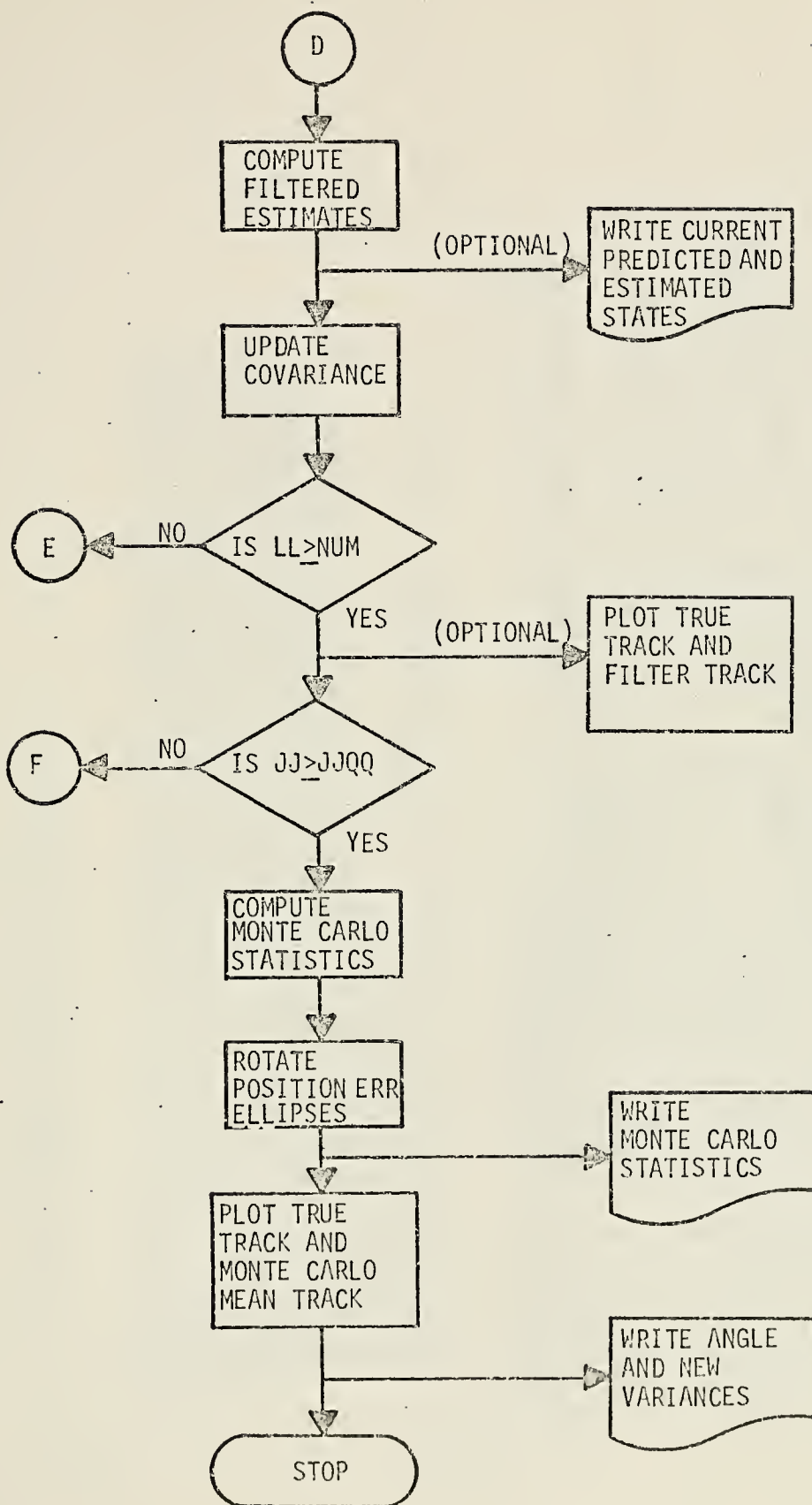


Flow Chart for X-Y Filter, Filter Program










```

1102 FORMAT('O XINT =',F15.2,10X,'YINT =',F15.2)
2000 FFORMAT('O PRED BEARING',F15.2,10X,'PRED FREQ1',F15.3,10X,
1 FFORMAT FREQ2',F15.3,10X,'PRED FREQ3',F15.3)
2010 FFORMAT('O MEAS BEARING',F15.2,10X,'MEAS FREQ1',F15.3,10X,
1 FFORMAT FREQ2',F15.3,10X,'MEAS FREQ3',F15.3)
2011 FFORMAT('O RESIDUALS - ANGLE',F10.2,10X,'FREQ1',F10.3,10X,
1 FFORMAT FREQ2',F10.3,10X,'FREQ3',F10.3)
5000 FFORMAT('O SIGMA2 = ',F10.8)
5001 FFORMAT('O SIGMA2 = ',F10.8)
5002 FFORMAT('O SIGMA2 = ',F10.8)
5003 FFORMAT('O SIGMA2 = ',F10.8)
5004 FFORMAT('O SIGMA2 = ',F10.8)
5005 FFORMAT('O SIGMA2 = ',F10.8)
5006 FFORMAT('O SIGMA2 = ',F10.8)
5007 FFORMAT('O SIGMA2 = ',F10.8)
5008 FFORMAT('O SIGMA2 = ',F10.8)
5009 FFORMAT('O SIGMA2 = ',F10.8)
5010 FFORMAT('O SIGMA2 = ',F10.8)
6000 FFORMAT('O SIGMA2 = ',F10.8)
6001 FFORMAT('O SIGMA2 = ',F10.8)
6002 FFORMAT('O SIGMA2 = ',F10.8)
6003 FFORMAT('O SIGMA2 = ',F10.8)
6004 FFORMAT('O SIGMA2 = ',F10.8)
6005 FFORMAT('O SIGMA2 = ',F10.8)
6006 FFORMAT('O SIGMA2 = ',F10.8)
6007 FFORMAT('O SIGMA2 = ',F10.8)
6008 FFORMAT('O SIGMA2 = ',F10.8)
6009 FFORMAT('O SIGMA2 = ',F10.8)
6010 FFORMAT('O SIGMA2 = ',F10.8)
8000 FFORMAT('O SIGMA2 = ',F10.8)
9010 FFORMAT('O SIGMA2 = ',F10.8)

```

```

CCCCC CCCCC CCCC C
TIME IS IN SECONDS
IXA AND IXF ARE STARTING NUMBERS FOR RANDOM NUMBER SUBROUTINE.
READ(5,5004) IXA,IXF
NSPOT IS THE NUMBER OF SEPERATE TRACKS TO BE RUN. SUBX, SUBY, AND
SUBVEL, AND SUBHD ARE STARTING POSITION OF TARGET, VELOCITY AND
HEADING FOR THE TRACK.
READ(5,6000) NSPOT
READ(5,6001) (SUBX(I),SUBVEL(I),SUBHD(I),I=1,NSPOT)
JJQC IS THE NUMBER OF SIMULATIONS PER TRACK.
READ(5,6000) JJQC
DC 204 JZA=1,5
DC 204 M=2,4
JZA=1
WRITE(6,5005) IXA,IXF
WRITE(6,6003) NSPOT
WRITE(6,6001) (SUBX(I),SUBVEL(I),SUBHD(I),I=1,NSPOT)
WRITE(6,6002) JJQC

```



```

C C      NUMBER OF TIME POINTS IS NUM
C C      DATA NUM,TINT/20,100./
C C      DATA X(1),X(2),X(3),X(4),X(5)/0.,0.,0.,0.,4500.,-1500./
C C      DATA Y(1),Y(2),Y(3),Y(4),Y(5)/0.,0.,0.,8000.,-4000./
C C      X(2) = 2000.
C C      X(2) = -2000.
C C      X(3) = 2000.
C C      X(3) = 2000.
C C      Y(4) = 7000.
C C      Y(5) = -5000.
C C      T(1) = 0.0
C C      DO 9 KL=2,NUM
C C      9 T(KL) = T(KL-1) + TINT
C C      WRITE(6,5010) NUM,(T(I),I=1,NUM)
C C      NUM1 = NUM + 1
C C      THE SENSORS ARE POSITIONED AT X(I), Y(I).
C C      DO 204 NS=1,NSPOT
C C      INITIAL SUB POSITION    XSO,YSO
C C      SUB SPEED AND DIRECTION ARE VSS AND THTA
C C      XSO = SUBX(NS)
C C      YSO = SUBY(NS)
C C      VSS = SUBVEL(NS)
C C      THTA = SUBHD(NS)
C C      TO = T(1)
C C      CONVERT FROM DEGREES TO RADIAN
C C      THTRAS = THTA/57.29578
C C      CONVERT FROM KNOTS TO YARDS PER SECOND
C C      VS = VSS*.563
C C      FREQ = 500.
C C      VNED = 1640.
C C      NUMBER OF OBSERVATION VARIABLES IS M.
C C      THIS DETERMINES THE NUMBER OF ROWS OF HOB.
C C      M1 = M - 1
C C      M2 = M * 5
C C      MM = M * M

```



```

C      WRITE(6,62) M1
C      DO 1 I=1,NUM
C      XS(I) = XSO + VS*COS(THTAS)*(T(I) - TO)
C      YS(I) = YSO + VS*SIN(THTAS)*(T(I) - TO)
C      LOOP FOR EACH HYDROPHONE
C      DO 2 J=1,M1
C      - BR(J,I) = ((XS(I) - X(J))**2 + (YS(I) - Y(J))**2)**.5
C      - IF((XS(I) - X(J)).GE.0.0) GO TO 5
C      - IF((YS(I) - Y(J)).GE.0.0) BR(J,I) = BR(J,I) + 3.14159
C      - IF((YS(I) - Y(J)).LT.0.0) BR(J,I) = BR(J,I) - 3.14159
5      CONTINUE
C      VR(J) = VS*COS(THTAS - BR(J,I)) / VR(J)/VMED)
C      - FD(J,I) = FREQ/((1.0 - VR(J)/VMED))
C      - VRDOT(J) = -(VS*SIN(THTAS - BR(J,I))**2/R(J,I)
C      FDDOT(J,I) = (FD(J,I)*FD(J,I)*VRDOT(J))/(FREQ*VMED)
C      CONVERT FROM RADIANS TO DEGREES FOR OUTPUT
C      BR(J,I) = BR(J,I)*57.29578
2      CONTINUE
C      XSO = XS(I)
C      YSO = YS(I)
C      TO = T(I)
1      CONTINUE
C      OUTPUT STATEMENTS FOR BUOYS. REMOVE COMMENT SYMBOL AS DESIRED.
C      WRITE(6,99)
C      DO 4 L=1,M1
C      WRITE(6,100) L,X(L),Y(L)
C      DO 4 K=1,NUM
C      WRITE(6,300) T(K),R(L,K),BR(L,K),FD(L,K),FDDOT(L,K),XS(K),YS(K)
4      CONTINUE
C      SET OF PARAMETERS FOR CALL GAUSS SUBROUTINE
C      BEARING STANDARD DEVIATION IS 5 DEGREES
C      SA = 5.
C      IF(JZA.EQ.2) SA = 2.0
C      IF(JZA.EQ.3) SA = 8.0
C      IF(JZA.EQ.4) SA = 11.0
C      IF(JZA.EQ.5) SA = 14.0

```



```

C      WRITE(6,8000) SA
C      AMA = 0.
C
C      FREQUENCY STANDARD DEVIATION IS .04 HZ
C
C      SF = .04
C      AMF = 0.
C      DC 398 IY=1,20
C      DC 399 IX=1,5
C      XMC{IX,IY} = 0.
C      XYVAR{IX,IY} = 0.
C      XVAR{IX,IY} = 0.
C      XMN{IX,IY} = 0.
C      YMN{IX,IY} = 0.
C      CCNTINUE
C      XINT{2} = 0.
C      YINT{2} = 0.
C      XINVE = 0.
C      YINVE = 0.
C      DC 397 JJ=1,20
C      RXY{JJ} = 0.
C      XYA{JJ} = 0.
C      RVXY{JJ} = 0.
C      VXVA{JJ} = 0.
C      CCNTINUE
C      AVKJ = 0.
C      WRITE(6,400) VSS,THTA
C
C      INITIAL ESTIMATE AT TARGET VELOCITY IS AVSK, IN KNOTS.
C      AVSK = 6.0
C      WRITE(6,711) AVSK
C
C      THIS DO LOOP RUNS JJQQ SIMULATIONS PER TRACK.
C      SET MEASUREMENT MATRIX Z TO ZERO.
C
C      DC 44 JZ=1,100
C      DC 44 KZ=1,5
C      Z{JZ,KZ} = 0.0
C
C      AVEVA = 0.
C      VARVA = 0.
C      WRITE(6,1092)
C      CCVXX = 0.0
C      CCVXY = 0.0
C      CCVYY = 0.0

```



```

CC CC CC CC
DC 201 JQ=1, JJQQ
IRE = 0
DATA F(1), F(2), F(3)/0., 0., 0./
WRITE(6, 59)
SIMULATE BUOY MEASUREMENTS.
DC 50 K=1, NUM
CALL GAUSS(IXA, SA, AMA, VA)
AVEVA = AVEVA + VA
VARVA = VARVA + VA**2
Z(K, 1) = BR(1, K) + VA
CALL GAUSS(IXF, SF, AMF, VF)
Z(K, 2) = FD(1, K) + VF
IF(M.EQ.2) GO TO 490
CALL GAUSS(IXF, SF, AMF, VF2)
Z(K, 3) = FD(2, K) + VF2
IF(M.EQ.3) GO TO 490
CALL GAUSS(IXF, SF, AMF, VF3)
Z(K, 4) = FD(3, K) + VF3
CONTINUE
490 WRITE(6, 60) T(K), BR(1, K), VA, Z(K, 1)
WRITE(6, 61) FD(1, K), VF, Z(K, 2), FD(2, K), VF2, Z(K, 3), VF3, Z(K, 4)
50 CONTINUE
CC CC CC CC CC CC CC CC

```

KALMAN FILTER, LINEAR STATE S USING X, Y COORDINATES
WITH NON - LINEAR MEASUREMENT MATRIX AND STATE DEPENDENT
EXCITATION

CLEAR ALL MATRICES TO ZERO.

```

DC 11 JJ=1, 25
COV(JJ) = 0.
COVK(JJ) = 0.
PHI(JJ) = 0.
PHITR(JJ) = 0.
QERR(JJ) = 0.
RERR(JJ) = 0.
BERR(JJ) = 0.
TEMP(JJ) = 0.
CONTINUE
11 DC 12 JJ=1, 20
GAIN(JJ) = 0.
GAINTR(JJ) = 0.
HCB(JJ) = 0.

```



```

CC      HCBTR(JJ) = 0.
CC      TEMP1(JJ) = 0.
CC      TEMP2(JJ) = 0.
CC      TEMP3(JJ) = 0.
12 CONTINUE

CC      DO 92 I=1,16
92 ZNOIS(I) = 0.

CC      BEARING STANDARD DEVIATION IS SQ RT OF ZNOIS(1) (IN RADIANS)
CC      ZNOIS(1) = (SA/57.29578)**2
CC      FREQUENCY STANDARD DEVIATION IS SQ RT OF ZNGIS(4)
CC      ZNOIS(M+2) = SF**2
CC      ZNGIS(MM) = SF**2
CC      IF(M.EQ.4) ZNOIS(11) = ZNOIS(MM)
CC      WRITE(6,1058)
CC      LPB = 1
CC      DO 96 LP=1,M
96 LPB = LP**M
CC      WRITE(6,1059) (ZNOIS(KB),KB=LPB,LPE)
CC      LPB = LPE + 1
CC      X(1) = X COORDINATE
CC      X(2) = X VELOCITY
CC      X(3) = Y COORDINATE
CC      X(4) = Y VELOCITY
CC      X(5) = REST FREQ, FO.

CC      ALL DIMENSIONS ARE GIVEN IN YARDS
CC      1 KNOT = .563 YARDS / SEC.
CC      MATRICES ARE STORED COLUMN BY COLUMN
CC      TO WRITE OUT MATRICES ROW BY ROW THE TRANSPOSE MUST BE WRITTEN
CC      SINCE THE STORAGE IN MEMORY IS BY COLUMN
CC      ESTIMATE OF SOUND VELOCITY IS VP.
CC      VP = 1640.
CC      AVS = AVSK*.563

```



```

XINVE = XINT(1)**2 + XINVE
YINVE = YINT(1)**2 + YINVE
CVX = VS*CS(THIAS)
CVY = VS*SIN(THIAS)
WRITE(6,1101) ARHALF, ARTRUE, XCUR(2), DVX, XCUR(4), OVY
WRITE(6,1101) ATHETH, HDT, HDING, FQ
ARHALF = ARHALF/1000.

```

CC CC CC

STANDARD DEVIATION OF INITIAL VELOCITY ASSUMED TO BE 3 KNOTS
(1.7 YDS/SEC)

```

ATHETH = ATHETH/RAD
ADELTH = ADELTH/RAD
AK = FLOAT(KJ)

```

CC

```

DIAGONAL INITIAL COVARIANCE IF M GE 3
CCVK(1) = 4.0
CCVK(7) = 2.0
CCVK(13) = 4.0
CCVK(19) = 2.0
CCVK(25) = 1.0
IF(M.GE.3) GO TO 149
INITIAL COVARIANCE MATRIX

```

CC

```

CST = COS(ATHETH)
SNT = SIN(ATHETH)
FF = 1.4142*SF
AA = 1.4142*SA/RAD
VV = 3.563
AVAV = (SA/SQRT(AK))/RAD
HUF = HEAD(ADELFF+FF, ADELTH, AVS, ATHETH)
HDF = HEAD(ADELF-FF, ADELTH, AVS, ATHETH)
HDA = HEAD(ADELF, ADELTH+AA, AVS, ATHETH)
HUV = HEAD(ADELF, ADELTH-AA, AVS, ATHETH)
HDV = HEAD(ADELF, ADELTH, AVS+VV, ATHETH)
XDF = RNC(AVS, ADELTH, HUF, ATHETH)*CST
SGXF = 5*(XUF-XDF)
XUA = RNC(AVS, ADELTH+AA, HUA, ATHETH)*CST
XDA = RNC(AVS, ADELTH-AA, HDA, ATHETH)*CST
SGXA = 5*(XUA-XDA)
XUV = RNC(AVS+VV, ADELTH, HUV, ATHETH)*CST
XDV = RNC(AVS-VV, ADELTH, HDV, ATHETH)*CST
SGXV = 5*(XUV-XDV)
XUI = ARHALF*COS(ATHETH+AVAV)
XDI = ARHALF*COS(ATHETH-AVAV)

```



```

149 CONTINUE
WRITE(6,1061) COVK
PARAMETERS FOR EXCITATION MATRIX          SIGTH -- HEADING STD DEV.
COEF SQUARED FOR VARIANCE                SIGVS -- VELOCITY STD DEV.
                                           SIGFO -- REST FREQ STD DEV.
SIGTH = 10. DEGREES/100. SEC   $(10 \pi / 180)^2 \times 100$ 
SIGTH = 3.04
SIGVS = 1 KNOT/100. SEC   $(.5143)^2 \times 100 = \sigma_{VS} = 3.1, y_{15}$ 
SIGVS = 31.7
SIGFO = .1 HZ/100. SEC   $(.1)^2 \times 100 = \sigma_F^2$ 
SIGFO = 1.0

LOOP FOR MEASUREMENT DATA

TT = -ATHALF/1000.
DO 200 LL=1,NUM
IF(LL.EQ.1) GO TO 88
TT=(T(LL) - T(LL-1))/1000.

PUT IN PHI MATRIX
88 PHI(1) = 1.
   PHI(7) = 1.
   PHI(13) = 1.
   PHI(19) = 1.
   PHI(25) = 1.
   PHI(6) = TT
   PHI(18) = TT
   CALL GMTRA(PHI,PHITR,5,5)
   WRITE(6,1000) T(LL)

WRITE(6,1001) PHITR
CALL GMPRD(PHI,XCUR,XPRED,5,5,1)

CALCULATE PREDICTED COVARIANCE MATRIX
NEED POSITIVE VALUES IN PHI MATRIX FOR COVARIANCE UPDATE.
PHI(6) = ABS(TT)
PHI(18) = ABS(TT)
PHITR(2) = PHI(6)
PHITR(14) = PHI(18)

CALCULATION OF THE STATE EXCITATION MATRIX. IT IS STATE DEPENDENT.
VSX = XCUR(2)**2 + XCUR(4)**2

```



```

AX = (XCUR(2)**2/VSX)*SIGVS + (XCUR(4)**2)*SIGTH
BX = (XCUR(2)**XCUR(4))* (SIGVS/VSX - SIGTH)
CX = (XCUR(4))* (SIGVS + (XCUR(2)**2)*SIGTH)
QEXX(1) = .25*TT**3
QEXX(2) = .25*TT**3
QEXX(3) = .25*TT**3
QEXX(4) = .25*TT**3
QEXX(5) = .25*TT**3
QEXX(6) = .25*TT**3
QEXX(7) = .25*TT**3
QEXX(8) = .25*TT**3
QEXX(9) = .25*TT**3
QEXX(10) = .25*TT**3
QEXX(11) = .25*TT**3
QEXX(12) = .25*TT**3
QEXX(13) = .25*TT**3
QEXX(14) = .25*TT**3
QEXX(15) = .25*TT**3
QEXX(16) = .25*TT**3
QEXX(17) = .25*TT**3
QEXX(18) = .25*TT**3
QEXX(19) = .25*TT**3
QEXX(20) = .25*TT**3
QEXX(21) = .25*TT**3
QEXX(22) = .25*TT**3
QEXX(23) = .25*TT**3
QEXX(24) = .25*TT**3
QEXX(25) = .25*TT**3

```

C

```

CALL GMPRD(PHI,COVK,TEMP4,5,5,5)
CALL GMPRD(TEMP4,PHITR,TEMP5,5,5,5)
DO 7 JJ=1,25
  CCV(JJ) = TEMP5(JJ) + QEXIT(JJ)
  WRITE(6,1060) CCV
1060 FORMAT(1X,25F10.5)

```

7

CCCCCCCCCCCC

```

LINEARIZED OBSERVATION MATRIX (TRANPOSE)
HOBTR(1) = HOBTR(5)
HOBTR(6) = HOBTR(10)
HOBTR(11) = HOBTR(15)

```

PREDICTED BEARING IS H1.

```

H1 = ATAN(XPRED(3)/XPRED(1))
IF(XPRED(1).GT.0.) GO TO 45
IF(XPRED(3).GE.0.) H1 = H1 + 3.14159
IF(XPRED(3).LT.0.) H1 = H1 - 3.14159
45 CONTINUE

```

45

C

```

DC 46 N=1,M1
NN = 5*N
XDIS = XPRED(1) - X(N)/1000.
YDIS = XPRED(3) - Y(N)/1000.
YVSQ(N) = XDIS**2 + YDIS**2
X(N) = XPRED(5)*VP/((XPRED(2)*XDIS + XPRED(4)*YDIS)/(XYSQ(N))*

```



```

1*-5)))
A = -(F(N)**2)/(XPRED(5)*VP)
HCBTR(NN+1) = (A*YDIS*(YDIS*XPRED(2) - XDIS*XPRED(4)))/XYSQ(N)**1.5
HCBTR(NN+2) = (A*XDIS)/XYSQ(N)**.5
HCBTR(NN+3) = -HOBTR(NN+1)*XDIS/YDIS
HCBTR(NN+4) = HOBTR(NN+2)*YDIS/XDIS
HCBTR(NN+5) = F(N)/XPRED(5)
CONTINUE
46 HOBTR(1) = -XPRED(3)/XYSQ(1)
HOBTR(2) = 0
HOBTR(3) = XPRED(1)/XYSQ(1)
HOBTR(4) = 0
HOBTR(5) = 0
CALL GMTRA (HOBTR,HOB,5,M)
WRITE(6,1020)(HOBTR(K),K=1,M2)

CCCC
CALCULATE THE GAIN MATRIX
CALL GMPRD(COV,HOBTR,TEMP1,5,5,M)
CALL GMPRD(HOB,TEMP1,TEMP2,M,5,M)
DO 22 JJ=1,M
22 TEMP2(JJ) = TEMP2(JJ) + ZNDIS(JJ)
CALL MINV(TEMP2,M,D,LT,WM)
WRITE(6,9010) D
CALL GMPRD(TEMP1,TEMP2,GAIN,5,M,M)
CALL GMTRA(GAIN,GAINTR,5,M)
WRITE(6,1039)
IB = 1
DO 47 I=1,5
47 IE = M*I
WRITE(6,1040) (GAINTR(MG),MG=IB,IE)
IB = IE + 1
CCCC
UPDATE THE COVARIANCE MATRIX
CALL GMPRD(GAIN,HOB,TEMP4,5,M,5)
DO 3 JJ=1,25
3 TEMP4(JJ) = -TEMP4(JJ)
DO 444 JJ=1,25,6
444 TEMP4(JJ) = 1.0 / TEMP4(JJ)
CALL GMPRD(TEMP4,COV,COVK,5,5,5)
WRITE(6,1070) COVK
CCCC
CALCULATE RESIDUAL BASED ON ESTIMATED VALUE OF THE OBSERVATIONS
AND THE INPUT MEASUREMENT
HH1 = H1*57.29578
RESA = Z(LL,1) - HH1

```



```

199 DC 199 KK=1,5
200 XCUR(KK) = XFIL(KK)
201 CONTINUE
    COVK(1)/JJQQ + COVXX
    COVK(3)/JJQQ + COVXY
    COVK(13)/JJQQ + COVYY
    WRITE(6,1093) XFIL,IRE
    WRITE(6,330)
    CALL PLOTP(X,Y,-5,1)
    CALL PLOTP(XINT,YINT,2,2)
    CALL PLOTP(XS,YS,NUM,2)
    CALL PLOTP(XSTAR,YSTAR,NUM,3)
    CONTINUE
    AVEVA = AVEVA/(JJQQ*NUM)
    VARVA = (VARVA/(JJQQ*NUM))**.5
    WRITE(6,66) AVEVA,VARVA
    WRITE(6,712) AVKJ
    WRITE(6,701) JJQQ
    DO 203 LL=1,NUM
    DC 202 J=1,5
    XMC(J,LL) = XMC(J,LL)/JJQQ
    XVAR(J,LL) = (XVAR(J,LL))**.5
    XSTD(J,LL) = XVAR(J,LL)**.5
    XYVAR(1,LL) = (XYVAR(1,LL)) - JJQQ*XMC(1,LL)/(JJQQ-1)
    XYVAR(2,LL) = (XYVAR(2,LL)) - JJQQ*XMC(2,LL)/(JJQQ-1)
    RXY(LL) = XYVAR(1,LL)/(XSTD(1,LL))
    RVXYV(LL) = XYVAR(2,LL)/(XSTD(2,LL))
    JJQQ*XMC(J,LL)**2)/(JJQQ-1)

    CALCULATE ROTATION ANGLE OF COVARIANCE ELLIPSES AND
    THE NEW VARIANCES ALONG THE COORDINATES.

    XAY = 2.*XYVAR(1,LL)/(XVAR(1,LL) - XVAR(3,LL))
    AAI1 = ATAN(XAY)
    XVA(LL) = AAI1*RAD/2.
    SG1 = .5*(XVAR(1,LL) + XVAR(3,LL))
    SG2 = XYVAR(1,LL)/SIN(AAI1)
    SNEWXV = SG1 - SG2
    SNEWXS = SQR(T(SNEWXV))
    SNEWYS = 2.*XYVAR(2,LL)/(XVAR(2,LL) - XVAR(4,LL))
    VB11 = ATAN(VXAVY)
    VXVYA(LL) = BB11*RAD/2.
    SG3 = .5*(XVAR(2,LL) + XVAR(4,LL))
    SG4 = XYVAR(2,LL)/SIN(BB11)
    SNEVXV = SG3 - SG4
    SNEWXS = SQR(T(SNEVXV))

```



```

SNEVYS = SQRT(SNEVYV)
XMC(5,LL) = XMC(5,LL) + FREQ
WRITE(6,702) LL,(XMC(J,LL),J=1,5)
WRITE(6,703) (XVAR(J,LL),J=1,5)
WRITE(6,704) (XSTD(J,LL),J=1,5)
WRITE(6,705) (XYVAR(1,LL),XYVAR(2,LL))
WRITE(6,706) (RXY(LL),RVXYV(LL))
WRITE(6,707) (XYA(LL),VXYVA(LL))
WRITE(6,709) SNEVXV, SNEVYV, SNEVXV, SNEVYV
WRITE(6,710) SNEVXS, SNEVYS, SNEVXS, SNEVYS
XMN(LL) = XMC(1,LL)*1000.
YMN(LL) = XMC(3,LL)*1000.
CALCULATE(TITLE, OF AVERAGED FINAL COVARIANCE MATRIX)
AXYD = ATAN(2.*COVXY/(COVXX - COVYY))
AXYD = AXYD*PI*180./5
SIGGY = 5*(COVXX + COVYY)
SCGGY = COVXY/SIN(AXY)
XXNEW = SIGGY + SOGGY
XXNEWS = SQRT(XXNEW)
YYNEWS = SQRT(YYNEW)
WRITE(6,715) COVXX, COVXY, COVXY, COVXX, XXNEW, YYNEWS
WRITE(6,716) AXYD, XXNEW, YYNEWS

C
WRITE(6,700) JJQQ
XINT(1) = 0.
XINT(2) = XINT(2)*1000./JJQQ
YINT(2) = YINT(2)*1000./JJQQ
WRITE(6,5006) Y, Y5, 1)
CALL PLOTIP(XINT, Y, Y5, 1)
CALL PLOTIP(XS, YS, YMN, 2)
CALL PLOTIP(XMN, YMN, 3)
CALL PPYF(NUM)
CCNT INUE
END
FUNCTION HEAD(DF, DT, VS, AV)
FORMAT('0', ASINE, 'F15.4')
COMMON/BLK1/ FQ, VP, FO, PI, RAD
WRITE(6,8001) DF, DT, VS, AV, FQ, VP, FO, PI, RAD
ASINE = 1.0
WRITE(6,10) ASINE
IF(ASINE.GT.-1.) ASINE = -1.
IF(ASINE.LT.-1.) ASINE = 1.
IF(FQ.LT.FO) H = AV + ARSIN(ASINE)

```



```

IF(H.GT.PI) H = H - 2.*PI
IF(H.LT.-PI) H = H + 2.*PI
HEAD = H
RETURN
END
SUBROUTINE PPFY(NUM)
COMMON XS(100),YPT(20),X(3),Y(3),YM(20),M1
DIMENSION XPT(20),YPT(20),X(3),Y(3),YM(20),M1
REAL XTITLE(9),MON(1),RUN(1),E(1),CA(1),SIMU(1),LATI(1),
1,CON(1),T(5),MITS(1),CHAN(1),GFI(1),ALTER(1),OUT(1),
REAL GEOIN(5),MITS(1),IN(1),S(20),QOY(1),DS(1)
REAL AXIS INCREMENT IS DEX AND DEY
DEY = 1333.
DEX = 1333.
X(1) = 0.
X(2) = 1.
X(3) = 1.
Y(1) = 0.
Y(2) = 1.
Y(3) = 1.
CALL SYMBOL(0,0,0,35,MIT,0,0,20)
CALL PLG(0,0,0,35,MIT,0,0,20)
CALL AXIS(0,0,0,35,MIT,0,0,20)
CALL AXIS(0,0,0,35,MIT,0,0,20)
CALL AXIS(0,0,0,35,MIT,0,0,20)
CALL AXIS(0,0,0,35,MIT,0,0,20)
CALL LINE(X,Y,M1,1,4)
CALL LINE(X,Y,M1,1,4)
DRAW KK=1, XS(KK)/DEX
XPT(KK) = YS(KK)/DEY
CONTINUE
CALL LINE(XPT,YPT,NUM,1,3)
CALL LINE(XPT,YPT,NUM,1,3)
DRAW KK=1, X(MN(J))/DEX
XPT(J) = Y(MN(J))/DEY
CONTINUE
DRAW OUTPUT NUM,1,-5)
CALL LINE(XPT,YPT,NUM,1,3)
CALL LINE(XPT,YPT,NUM,1,3)
CALL SYMBOL(0,0,0,35,MIT,0,0,20)

```



```

CALL PLOT(-3.50,6.5,-3)
CALL PLOTE
RETURN
END
//GC.SYSIN DD *
320581 630487
1 1000. 6000. 8.0 -70.
200

```



```

334 FORMAT('1',FREQ1,RESIDUALS,MEAN',F10.3,',2ND MD',F10.3,',
1 STD DEV',F10.3)
335 1 FORMAT('0'+FREQ2,RESIDUALS,MEAN',F10.3,',2ND MD',F10.3,',
1 STD DEV',F10.3//)
400 FCRMAT('00,INITIAL,SPEED IS',F6.2,',KNOTS,HEADING IS',F8.2)
401 FCRMAT('00,INITIAL,FILTER STATES',F10.2,8X,'RCPA',F10.2,8X,'XCPA',F10.2
1,8X,'TAR HD',F10.2,8X,'REST FREQ',F10.3)
666 1 FCRMAT('00,KJ',F10.3,5X,'Z(1,1)',F10.3,
15X,'ADELTH',F10.3)
667 FCRMAT('00,TIME,KJ',F10.2,5X,'Z(KJ,2)',F10.2,5X,'Z(1,2)',F10.2,
1 FCRMAT('00,ADELF',F10.2,5X,'F10.2)
668 FCRMAT('00,ASINE',F10.6,5X,'F10.6)
669 1 FCRMAT('00,COS(THS-TH2)',F15.5,10X,'THS-TH2',F15.2,10X,
1,TH2AVE',F15.2)
670 FCRMAT('00,INITIAL,RANGE USING 2 ND BUOY',F15.2)
700 FCRMAT('00,MONTE CARLO PLOT BASED ON',F10.1,RUNS'//)
701 1 FCRMAT('00,MONTE CARLO STATISTICS ON',F10.1,RUNS'//4X,TIME POIN
1,1,16X,'Y',F15.3,5X,'V',F15.3,5X,'TH',F15.3,5X,'FC'//)
702 FCRMAT('00,110,10X,10X,5F15.3)
703 FCRMAT('00,110,10X,10X,5F15.3)
704 FCRMAT('00,STD DEV',F15.3,5X,'11X,8X,2F15.3)
705 FCRMAT('00,CORR',F15.3,5X,'9X,2F15.3)
706 FCRMAT('00,CORR',F15.3,5X,'9X,2F15.3)
707 FCRMAT('00,CORR',F15.3,5X,'9X,2F15.3)
708 FCRMAT('00,CORR',F15.3,5X,'9X,2F15.3)
709 FCRMAT('00,CORR',F15.3,5X,'9X,2F15.3)
710 FCRMAT('00,CORR',F15.3,5X,'9X,2F15.3)
711 FCRMAT('00,CORR',F15.3,5X,'9X,2F15.3)
1000 FCRMAT('00,INITIAL,TIME',F7.1,10X,'F7.1//)
1001 FCRMAT('00,PHI MD',F13.5X,5X,'RANGE',F8.3,5X,'BEARING',F8.3,5X,'TAR R
1002 1 FCRMAT('00,BUOY',F8.3,5X,'TAR BEAR',F8.3,5X,'FREQ',F8.3)
1020 FCRMAT('00,10X,5X,5F15.3)
1039 FCRMAT('00,10X,5X,5F15.3)
1040 FCRMAT('00,10X,5X,5F15.3)
1058 FCRMAT('00,10X,5X,5F15.3)
1059 FCRMAT('00,10X,5X,5F15.3)
1060 FCRMAT('00,10X,5X,5F15.3)
1061 FCRMAT('00,10X,5X,5F15.3)
1070 FCRMAT('00,10X,5X,5F15.3)
1076 FCRMAT('00,10X,5X,5F15.3)
1077 FCRMAT('00,10X,5X,5F15.3)
1078 FCRMAT('00,10X,5X,5F15.3)
1079 FCRMAT('00,10X,5X,5F15.3)
1080 FCRMAT('00,10X,5X,5F15.3)

```



```

1081 FORMAT('+',10I1,' IRE =',I3)
1082 1V,'F9.3) MEAN',F9.3,' 2ND MO',F9.3,' ST DE
1083 FREQ RESID DATA',6X,3F15.3/,24X,3F15.3)
1084 FORMATT(//) REINITIALIZE FILTER, RESIDUAL COUNT GR
1085 LEATER THAN 3 ***** REINITIALIZE FILTER, RESIDUAL COUNT GR
1086 FORMATT(//) REJECTED TRACK:////)
1087 FORMATT(//) RCPC -- BEFORE, F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA --- BEFOR
1088 1E',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
1089 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
1090 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
1091 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
1092 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
1093 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
1094 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
1102 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
15000 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
5001 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
5002 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
5003 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
5004 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
5005 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
5010 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
6000 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
6001 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
6002 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
6003 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
6004 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
9010 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
9500 1F',F8.3, 5X, 'AFTER', F8.3, 10X, 'XCPA', 13X, 'VS', 12X, 'THIAS', 10X,
DATA RAD,PI/57.29578,3.14159/
TIME IS IN SECONDS
IXA AND IXF ARE STARTING NUMBERS FOR RANDOM NUMBER SUBROUTINE.
READ(5,5004) IXA,IXF
NSPOT IS THE NUMBER OF SEPERATE TRACKS TO BE RUN. SUBX, SUBY, AND
SUBVEL, AND SUBHD ARE STARTING POSITION OF TARGET, VELOCITY AND
HEADING FOR THE TRACK.
READ(5,6000) NSPOT
READ(5,6001) (SUBX(I),SUBY(I),SUBVEL(I),SUBHD(I),I=1,NSPOT)

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C CCCCCC CCCCCC


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CC CC JJQQ IS THE NUMBER OF SIMULATIONS PER TRACK.
CC CC READ(5,60.00) JJQQ
CC CC DO 204 JLP=1,4
CC CC JLP = 1
CC CC WRITE(6,5005) IXA,IXF
CC CC WRITE(6,6003) NSPOT
CC CC WRITE(6,6001) (SUBX(I),SUBY(I),SUBVEL(I),SUBHD(I),I=1,NSPOT)
CC CC WRITE(6,6002) JJQQ
CC CC NUMBER OF TIME POINTS IS NUM
CC CC DATA NUM,TINT/20,100./
CC CC X(1) = 0.
CC CC X(2) = 2000.
CC CC X(3) = 2000.
CC CC X(4) = -1500.
CC CC X(5) = 4500.
CC CC Y(1) = 0.
CC CC Y(2) = 2000.
CC CC Y(3) = -2000.
CC CC Y(4) = 7000.
CC CC Y(5) = -5000.
CC CC T(1) = 0.0
CC CC DO 9 KL=2,NUM
CC CC T(KL) = T(KL-1) + TINT
CC CC WRITE(6,5010) NUM,{T(I),I=1,NUM}
CC CC NUM1 = NUM + 1
CC CC DO 204 NS=1,NSPOT
CC CC INITIAL SUB POSITION XSO,YSO
CC CC SUB SPEED AND DIRECTION ARE VSS AND THTA
CC CC XSO = SUBX(NS)
CC CC YSO = SUBY(NS)
CC CC VSS = SUBVEL(NS)
CC CC THTA = SUBHD(NS)
CC CC TO = T(1)
CC CC CONVERT FROM DEGREES TO RADIAN
CC CC THIAS = THTA/57.29578
CC CC CONVERT FROM KNOTS TO YARDS PER SECOND
CC CC VS = VSS*.563

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```

CC      WRITE(6,300) T(K),R(L,K),BR(L,K),FD(L,K),FDCOT(L,K),XS(K),YS(K)
4      CONTINUE
CC      CALCULATE TRUE VALUES OF RCPA AND TCPA.
      ANG = (THIA - BR(1,1))/57.29578
      REX = ABS(R(1,1)*SIN(ANG))
      XEXI = (R(1,1)**2 - REX**2)**.5/1000.
      IF(FD(1,1).GT.FREQ) XEXI = -XEXI
CC      M IS THE NUMBER OF OBSERVATION VARIABLES.
      M = 2*NDIFA + NLOFA
      M1 = M-1
      M2 = M*5
      MM = M*M
CC      SET OF PARAMETERS FOR CALL GAUSS SUBROUTINE
      BEARING STANDARD DEVIATION IS 5 DEGREES
      SA = 5.
      AMA = 0.
      FREQUENCY STANDARD DEVIATION IS .04 HZ
      SF = .04
      AMF = 0.
      DC 398 IX=1,20
      DC 399 IX=1,5
      XMC(IX, IY) = 0.
      XYVAR(IX, IY) = 0.
      XVAR(IX, IY) = 0.
      XMN(IY) = 0.
      YMN(IY) = 0.
      CCINTINUE = 0.
      XINT(2) = 0.
      YINT(2) = 0.
      XINVE = 0.
      YINVE = 0.
      DC 397 J1=1,20
      RXY(JJ) = 0.
      XYA(JJ) = 0.
      RVXVY(JJ) = 0.
      VXVYA(JJ) = 0.
      CCINTINUE
397

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```

CC      AVSK = 6.0
CC      WRITE(6,711) AVSK
CC      WRITE(6,400) VSS,THTA
CC      WRITE(6,63) NBU,NDIFA,NLOFA
CC      ZNOIS IS THE 'C' R 'C' MATRIX OF THE MEASUREMENT NOISE.
CC
93      DC 93 IQ=1,16
CC      ZNOIS(IQ) = 0.
CC
CC      BEARING STANDARD DEVIATION IS SQ RT OF ZNOIS(1) (IN RADIANS)
CC
CC      ZNCIS(1) = (SA/57.29578)**2
CC      IF(NDIFA.EQ.2) ZNOIS(11) = ZNOIS(1)
CC
CC      FREQUENCY STANDARD DEVIATION IS SQ RT OF ZNCIS(4)
CC
CC      ZNOIS(M+2) = SF**2
CC      ZNCIS(MM) = SF**2
CC      IF(INDIFA.EQ.1).AND.(NLOFA.EQ.2)) ZNOIS(11) = ZNOIS(MM)
CC
CC      WRITE(6,1058)
CC      LPB = 1 LP=1,M
CC      DC 96 LP=M
CC      LPE = LP*M
CC      WRITE(6,1059) (ZNOIS(KB),KB=LPB,LPE)
96      LPB = LPE + 1
CC      WRITE(6,1094)
CC      AVEVA = 0.
CC      VARVA = 0.
CC      THIS DO LOOP RUNS JJQQ SIMULATIONS PER TRACK.
CC
CC      DC 201 JQ=1,JJQQ
CC      SET Z MATRIX TO ZERO.
CC
CC      DO 49 JZ=1,100
CC      DC 49 KZ=1,6
49      Z(JZ,KZ) = 0.0
CC
CC      DATA F(1),F(2),F(3)/0.,0.,0./
CC      DATA THTAX(1),THTAX(2)/0.0,0.0/
CC      DATA TH(1),TH(2),TH(3)/0.0,0.0,0.0/
CC
CC      WRITE(6,59)
CC
CC      SIMULATE THE OBSERVATIONS.
CC
CC      DC 50 K=1,NUM

```



```

DC 51 KJ=1,NDIFA
KJ1 = (KJ-1)**2
CALL GAUSS(IXA,SA,AMA,VA)
AVEVA = AVEVA + VA
VARVA = VARVA + VA**2
Z(K,1+KJ1) = BR(KJ,K) + VA
WRITE(6,60) T(K),BR(KJ,K),VA,Z(K,1+KJ1)
51 CONTINUE
DO 52 KJ=1,NBU
KJJ = (KJ-1)*NDIFA
CALL GAUSS(IXF,SF,AMF,VF)
Z(K,2+KJJ) = FD(KJ,K) + VF
WRITE(6,61) FD(KJ,K),VF,Z(K,2+KJJ)
52 CONTINUE
50 CONTINUE

```

KALMAN FILTER ROUTINE USING CONSTANT PARAMETERS AS STATES.

QEXIT IS THE EXCITATION MATRIX.

IRE IS THE RESIDUAL COUNTER

3 NEW TRACKS IS THE MAX TO BE TRIED

IRE = 0

ICOUNT = 3

IEND = ICOUNT + 3

```

X(1) = RANGE AT CPA - YDS.
RANGE IN KILO YARDS
X(2) = RANGE TO CPA - KILO YDS NEGATIVE BEFORE, POSITIVE AFTER
X(3) = TARGET SPEED - YDS/SEC
X(4) = TARGET COURSE - DEGREES FROM X-AXIS.
X(5) = REST FREQUENCY - HZ.

```

ALL DIMENSIONS ARE GIVEN IN YARDS

1 KNOT = .563 YARDS / SEC.

MATRICES ARE STORED COLUMN BY COLUMN

TO WRITE OUT MATRICES ROW BY ROW THE TRANSPOSE MUST BE WRITTEN
SINCE THE STORAGE IN MEMORY IS BY COLUMN

ESTIMATE OF SOUND VELOCITY IS VP.


```

C      VP = 1640.
C      INITIAL ESTIMATE AT TARGET VELOCITY IS AVSK, IN KNOTS.
C      AVS = AVSK*.563
C      CALCULATION OF THE INITIAL CONDITIONS FOR THE FILTER.
C      TO INITIALIZE THE FILTER THE TWO BEARING MEASUREMENTS MUST
C      BE GREATER THAN 3 STD DEV OF THE NOISE APART.
C      KJ = 2
C      FO IS ONLY USED TO DECIDE UP OR DOWN DOPPLER FOR HEADING CALCULATION
87  CONTINUE
   FC = 500.
   W = 1.0
   IF(NDIFA.EQ.1) CALL INIT11(AVS,FO,XFIL,XIN,YIN,SA,NUN)
   IF(NDIFA.EQ.2) CALL INIT20(AVS,FO,XFIL,XIN,YIN,SA,NUN)
   XINT(2) = XIN + XINT(2)
   YINT(2) = YIN + YINT(2)
   THFIL(1) = Z(1,1)
   WRPTE(6,1102)XIN,YIN
   RCPAI = XFIL(1)
   XCPAI = XFIL(2)
   WRPTE(6,401)XFIL
   XFIL(4) = XFIL(4)/RAD
      CLEAR ALL MATRICES TO ZERO.
DC 11 JJ=1,25
CCV(JJ) = 0.
CCVK(JJ) = 0.
QEXIT(JJ) = 0.
TEMP3(JJ) = 0.
PHITR(JJ) = 0.
PHITR(JJ) = 0.
TEMP4(JJ) = 0.
CONTINUE
DC 12 JJ=1,20
GAIN(JJ) = 0.
GAINTR(JJ) = 0.
HCB(JJ) = 0.
HCBTR(JJ) = 0.
TEMP1(JJ) = 0.
TEMP2(JJ) = 0.
11

```



```

12 CONTINUE
13 DC 13 JJ=1,30
HCBTR(JJ) = 0.
13 CONTINUE
XEX = XEXI

C
C SET DIAGONAL INITIAL COVARIANCE IF NDIFA = 2
CCVK(1) = 4.0
CCVK(7) = 16.0
CCVK(13) = 2.68
CCVK(19) = .36
CCVK(25) = 1.0
IF(NDIFA.EQ.2) GO TO 149

C SINGLE BUOY INITIAL COVARIANCE MATRIX
C
FAVE = 0.
DC 130 JJ=1,KJ
130 FAVE = Z(JJ,2)/KJ + FAVE

C
C ADELTH = ADELTH/RAD
ATHETH = ATHETH/RAD
C CALCULATION OF INITIAL COVARIANCE MATRIX
C
ADELT = 2.*ATHALF/1000.
FF = 1.4142*SF
AA = 1.4142*SA/RAD
VV = 3.*.563
AK = FLOAT(KJ)
AVAV = (SA/SQRT(AK))/RAD

C
HUF = HEAD(ADELF+FF, ADELTH, AVS, ATHETH)
HDF = HEAD(ADELF-FF, ADELTH, AVS, ATHETH)
SGHF = 5*(HUF - HDF)
HUA = HEAD(ADELF, ADELTH+AA, AVS, ATHETH)
HDA = HEAD(ADELF, ADELTH-AA, AVS, ATHETH)
SGHA = 5*(HUA - HDA)
HUV = HEAD(ADELF, ADELTH, AVS+VV, ATHETH)
HCV = HEAD(ADELF, ADELTH, AVS-VV, ATHETH)
SGHV = 5*(HUV - HDV)
SGHT = AVAV
RUF = RCP(AVS, ADELTH, HUF, ATHETH)
RDF = RCP(AVS, ADELTH, HDF, ATHETH)
SGRF = 5*(RUF - RDF)
RUA = RCP(AVS, ADELTH+AA, HUA, ATHETH)
RDA = RCP(AVS, ADELTH-AA, HDA, ATHETH)
SGRA = 5*(RUA - RDA)
RUV = RCP(AVS+VV, ADELTH, HUV, ATHETH)

```



```

RDV = RCP(AVS-VV, ADELTH, HDV, ATHETH)
SGRV = .5*(RUV - RDV)
XUF = XCP(AVS, ADELTH, HUF, ATHETH)
XDF = XCP(AVS, ADELTH, HDF, ATHETH)
SGXF = .5*(XUF - XDF)
XUA = XCP(AVS, ADELTH+AA, HUA, ATHETH)
XDA = XCP(AVS, ADELTH-AA, HDA, ATHETH)
SGXA = .5*(XUA - XDA)
XUV = XCP(AVS+VV, ADELTH, HUV, ATHETH)
XDV = XCP(AVS-VV, ADELTH, HDV, ATHETH)
SGXV = .5*(XUV - XDV)
FUF = FCCP(AVS, HUF, ATHETH)
FDF = FCCP(AVS, HDF, ATHETH)
SGFF = .5*(FUF - FDF)
FUA = FCCP(AVS, HUA, ATHETH)
FDA = FCCP(AVS, HDA, ATHETH)
SGFA = .5*(FUA - FDA)
FUV = FCCP(AVS+VV, HUV, ATHETH)
FDV = FCCP(AVS-VV, HDV, ATHETH)
SGFV = .5*(FUV - FDV)

COVK(1) = SGRF**2 + SGRA**2 + SGRV**2
COVK(7) = SGXF**2 + SGXA**2 + SGXV**2
COVK(13) = VV**2
COVK(19) = SGHF**2 + SGHA**2 + SGHV**2 + SGHT**2
COVK(25) = SGFF**2 + SGFA**2 + SGFV**2
SET NO = 1 IF ONLY DIAGONAL TERMS ARE WANTED

NC = 1
IF (NO.EQ.1) GO TO 149
COVK(2) = SGRF*SGXF + SGRA*SGXA + SGRV*SGXV
COVK(3) = SGRV*VV
COVK(4) = SGRF*SGHF + SGRA*SGHA + SGRV*SGHV
COVK(5) = SGRF*SGFF + SGRA*SGFA + SGRV*SGFV
COVK(6) = COVK(2)
COVK(8) = SGXV*VV
COVK(9) = SGXF*SGHF + SGXA*SGHA + SGXV*SGHV
COVK(10) = SGXF*SGFF + SGXA*SGFA + SGXV*SGFV
COVK(11) = COVK(3)
COVK(12) = COVK(8)
COVK(14) = VV*SGHF
COVK(15) = VV*SGFV
COVK(16) = COVK(4)
COVK(17) = COVK(9)
COVK(18) = COVK(14)
COVK(20) = SGHF*SGFF + SGHA*SGFA + SGHV*SGFV
COVK(21) = COVK(5)
COVK(22) = COVK(10)

```

C

CC


```

C      COVK(23) = COVK(15)
C      COVK(24) = COVK(20)

C      149 CONTINUE
C      WRITE(6,1061) COVK
C
C      EXCITATION VALUES.
C
C      SIGTH      100 DEGREES/KILO SEC.      HEADING
C      SIGTH = 3.04
C      SIGVS      10 KNTS/KILO SEC.      SPEED
C      SIGVS = 31.7
C      SIGFO      .5 HZ/KILO SEC.      REST FREQ
C      SIGFO = .25
C
C      LCOOP FOR MEASUREMENT DATA
C
C      TO = AT HALF
C      IF(NDIFA.EQ.2) TO = T(1)
C      DO 200 LL=NDIFA,NUM
C      DELTAT = (T(LL) - TO)/1000.
C      DC 15 N=1,5
C      15 XPRED(N) = XFIL(N)
C      XPRED(1) = ABS(XPRED(1))
C      WRITE(6,1000) T(LL)
C
C      IF(LL.LE.KJ) GO TO 92
C      WDEL = 1.
C      DEL = THFIL(LL-1) * THFIL(1)
C      IF((DEL.LT.0.).OR.(DEL.GT.180.)) W = -1.0
C      92 CONTINUE
C      CALCULATE PREDICTED COVARIANCE MATRIX
C
C      PHI(1) = 1.
C      PHI(7) = 1.
C      PHI(12) = 1.
C      PHI(13) = 1.
C      PHI(19) = 1.
C      PHI(25) = 1.
C      CALL GMTRIA(PHI,PHITR,5,5)
C      NEED POSITIVE VALUES IN PHI MATRIX FOR COVARIANCE UPDATE.
C      PHI(12) = ABS(DELTAT)
C      PHITR(8) = PHI(12)
C      WRITE(6,1001) PHITR
C      CALL GMPRD(PHI,COVK,TEMP4,5,5,5)
C      CALL GMPRD(TEMP4,PHITR,COV,5,5,5)

```



```

C      QEXIT IS CALCULATED AT THE CURRENT STATE VALUE.
C
      QEXIT(1) = ((XPRED(2)*DELTA)**2)*SIGTH
      QEXIT(2) = -((XPRED(1))*XPRED(2)*DELTA**2)*SIGTH
      QEXIT(4) = W*XPRED(2)*DELTA**2*SIGTH
      QEXIT(6) = QEXIT(2)
      QEXIT(7) = ((-25*DELTA**2)*SIGVS
      QEXIT(8) = ((-W*XPRED(1))*DELTA**2)*SIGTH
      QEXIT(9) = ((-5*DELTA**3)*SIGVS
      QEXIT(12) = QEXIT(8)
      QEXIT(13) = DELTA**2*SIGVS
      QEXIT(16) = QEXIT(4)
      QEXIT(17) = QEXIT(9)
      QEXIT(19) = SIGTH*DELTA**2
      QEXIT(25) = SIGFO*DELTA**2

C      DO 7 JJ=1,25
C      COV(JJ) = COV(JJ) + QEXIT(JJ)
C
C      7 CONTINUE(6,1050) QEXIT
C      WRITE(6,1060) COV
C
      XPRED(2) = XPRED(2) + XPRED(3)*DELTA

C      LINEARIZED OBSERVATION MATRIX (TRANSPOSE)
      HOBTR(1) = HOBTR(5)      ANGLE
      HOBTR(6) = HOBTR(10)     FREQ(1)
      HOBTR(11) = HOBTR(15)    FREQ(2)
      HOBTR(16) = HOBTR(20)    FREQ(3)

C      DO 46 N=1,NBU
C      NN1 = (N-1)*10
C      IF(N.EQ.1) GO TO X(1)**2 + (Y(N) - Y(1))**2)**5/1000.
C      TH(N) = ATAN(Y(N) - Y(1))/(X(N) - X(1))
C      IF((X(N) - Y(1)).GE.0.) GO TO 48
C      IF((Y(N) - Y(1)).GE.0.) TH(N) = TH(N) + PI
C      IF((Y(N) - Y(1)).LT.0.) TH(N) = TH(N) - PI
C      ATHA = TH(N)
C      IF(ATHA.EQ.1) ATHA = 0.0
C      IF(CPA(N) = 1) XPRED(1) = W*D(N)*SIN(ATHA)
C      XCPA(N) = X*XPRED(2) + D(N)*COS(ATHA)
C      RANGE(N) = X*XPRED(2)**2 + XCPA(N)**2**5
C      THTAX(N) = XPRED(5)*VP/(VP + XPRED(3)*XCPA(N)/RANGE(N))
C      IF(XCPA(N) = 1) ATAN(W*RCPA(N)/XCPA(N))
C      IF(XCPA(N).LT.0) THTAX(N) = THTAX(N) - 2*PI
C      IF(THTAX(N).LT.-PI) THTAX(N) = THTAX(N) + 2*PI

```



```

C      ATH = TH(N)*RAD
      ATAR = THAX(N)*RAD
      WRITE(6,1002) N,D(N),ATH,RANGE(N),ATAR,F(N)
      HOBTR(1,N,N1) = -W*XCPA(N)/RANGE(N)**2
      HOBTR(2,N,N1) = W*RCPA(N)/RANGE(N)**2
      HOBTR(3,N,N1) = 0.0
      HOBTR(4,N,N1) = 1. + (D(N)/RANGE(N)**2)*(XCPA(N)*COS(ATHA) -
1W*RCPA(N)*SIN(ATHA) - D(N))
      HOBTR(5,N,N1) = 0.0
      FSQ = F(N)**2*XPRD(3)/(XPRD(5)*VP)
      HOBTR(6,N,N1) = FSQ*XCPA(N)**2/RANGE(N)**3
      HOBTR(7,N,N1) = -FSQ*RCPA(N)**2/RANGE(N)**3
      HOBTR(8,N,N1) = -FSQ*XCPA(N)/(XPRD(3)*RANGE(N))
      HOBTR(9,N,N1) = (FSQ*D(N)/RANGE(N))*(SIN(ATHA) - XCPA(N)*
1(W*XPRD(1)*COS(ATHA) + XPRD(2)*SIN(ATHA))/RANGE(N)**2)
      HOBTR(10,N,N1) = F(N)/XPRD(5)
      CONTINUE
46  IF(NDIFA.EQ.2) GO TO 55
      DO 54 NZ=1,5
      HOBTR(10+NZ) = HOBTR(15+NZ)
      HOBTR(15+NZ) = HOBTR(25+NZ)
      CONTINUE
54  CONTINUE
55  CALL GMTRA(HOBTR,HOB,5,M)
      WRITE(6,1020) (HOBTR(K),K=1,M2)

C      CALCULATE THE GAIN MATRIX

      CALL GMPRD(COV,HOBTR,TEMP1,5,5,M)
      CALL GMPRD(HOB,TEMP1,TEMP2,K,5,M)
      DO 22 JJ=1,M
      TEMP2(JJ) = TEMP2(JJ) + ZNOIS(JJ)
      CONTINUE
22  CALL MINV(TEMP2,M,DET,LT,WM)
      CALL MINV(TEMP2,M,DET,LT,WM)
      WRITE(6,9010) DET
      CALL GMPRD(TEMP1,TEMP2,GAIN,5,M,M)
      CALL GMTRA(GAIN,GAINTR,5,M)

C      PUT GAIN ARRAYS HERE FOR GAIN PLOT

      WRITE(6,1039)
      IB=1
      DO 47 I=1,5
      IE=M+I
      WRITE(6,1040) (GAINTR(MG),MG=IB,IE)
47  IB = IE + 1

C      UPDATE THE COVARIANCE MATRIX

```



```

C      CALL GMPRD(GAIN,HOB,TEMP3,5,M,5)
      DC 3 JJ=1,25
      TEMP3(JJ)=-TEMP3(JJ)
C      DC 44 JJ=1,25,6
      TEMP3(JJ)=1.+TEMP3(JJ)
C      CALL GMPRD(TEMP3,COV,COVK,5,5,5)
      WRITE(6,1070) COVK
C
C      CALCULATE THE RESIDUALS.
C
      WRITE(6,1076)
      ATAR1=THIAX(1)*RAD
      ATAR2=THIAX(2)*RAD
      TWO DIFAR=THIAX(1)*RAD-THIAX(2)*RAD
      RES1=Z(LL,1).GT.+.180.) RES1=RES1-360.
      IF(RES1.LT.-.180.) RES1=RES1+360.
      RES2=Z(LL,2)-F(1) THIAX(2)*RAD
      IF(RES2.LT.-.180.) RES2=RES2+360.
      IF(RES2.GT.+.180.) RES2=RES2-360.
      RES3=Z(LL,3)-F(1) THIAX(2)*RAD
      IF(RES3.LT.-.180.) RES3=RES3+360.
      IF(RES3.GT.+.180.) RES3=RES3-360.
      RES4=Z(LL,4)-F(2)
      IF(NDIFAR.EQ.2) GO TO 601
      ONE DIFAR AND TWO LOFAR.
      RES3=Z(LL,3)-F(3)
      IF(NLOFA.EQ.2) GO TO 602
      ONE DIFAR AND ONE LOFAR.
      RES4=0.0
      IF(NLOFA.EQ.1) GO TO 603
      ONE DIFAR AND NO LOFAR.
      RES3=0.0
      WRITE(6,1080) ATAR1,Z(LL,1),RES1,F(1),Z(LL,2),RES2
      GO TO 610
      WRITE(6,1077) ATAR1,Z(LL,1),RES1,F(1),Z(LL,2),RES2,ATAR2,Z(LL,3),
1 RES3,F(2),Z(LL,4),RES4
601 CONTINUE RES3/RAD
      IF(ABS(RES3).GT.2.*SA/RAD) IRE = IRE + 1
      GO TO 610
      WRITE(6,1078) ATAR1,Z(LL,1),RES1,F(1),Z(LL,2),RES2,F(2),Z(LL,3),
602 RES3,F(3),Z(LL,4),RES4
      CONTINUE
      IF(ABS(RES3).GT.25).OR.(ABS(RES4).GT.25) IRE = IRE + 1
      GO TO 610
      WRITE(6,1079) ATAR1,Z(LL,1),RES1,F(1),Z(LL,2),RES2,F(2),Z(LL,3),
1 RES3

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```

603  CONTINUE
    IF (ABS (RES3) .GT. .25) IRE = IRE + 1
610  CONTINUE
C
C      RES1 = RES1/RAD
C
C      IF (ABS (RES1) .GT. 2.*SA/RAD) IRE = IRE + 1
C      IF (ABS (RES2) .GE. 0.25) IRE = IRE + 1
C
C      SINGLE DIFA BUOY RESIDUAL STATISTICS
C
C      IF (NDIFA.EQ.2) GO TO 615
C
C      CALCULATE THE STATISTICS OF THE RESIDUALS
C      MEAN      SECOND MOMENT      STANDARD DEVIATION
C
C      AVRESA = AVRESA*(LL-1)/LL + RES1/LL
C      X22 = X22*(LL-1)/LL + RES1**2/LL
C      STDRA1 = X22**0.5
C      STDRA2 = (X22 - AVRESA**2)**0.5
C      AVRF1 = AVRF1*(LL-1)/LL + RES2/LL
C      X33 = X33*(LL-1)/LL + RES2**2/LL
C      STF11 = X33**0.5
C      STF12 = (X33 - AVRF1**2)**0.5
615  CONTINUE
C
C      IF YOU DONT WANT TO REINITIALIZE PUT      GO TO 88
C
C
C      GO TO 88
C      IF (IRE.LT. ICOUNT) GO TO 88
C      IF (ICOUNT.GE. IEND) GO TO 88
C      IF (IRE.GE. ICOUNT) KJ = LL
C      WRITE (6,1082) AVRESA,STDRA1,STDRA2
C      WRITE (6,1083) AVRF1,STF11,STF12,AVRF2,STF21,STF22
C      WRITE (6,1085)
C      CALL PLOTP(X,Y,-5,1)
C      CALL PLOTP(XINT,YINT,2,2)
C      CALL PLOTP(XS,YS,NUM,2)
C      NN11 = LL - 1
C      CALL PLOTP(XIAR,YIAR,NN11,3)
C      ICOUNT = ICOUNT + 1
C      IRE = 0
C      WRITE (6,1084)
C      GO TO 87
C
C      CONTINUE
C      DC 6 J=1,5
C      XFIL(J) = XPRED(J) + GAIN(J)*RES1 + GAIN(J+5)*RES2 + GAIN(J+10)*
C      1RES3 + GAIN(J+15)*RES4
88

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6  CCNTINUE = XFIL(5) - FREQ
DC 616 J=3,5
XMC(J,LL) = XFIL(J) + XMC(J,LL)
XVAR(J,LL) = XFIL(J)**2 + XVAR(J,LL)
616 CCNTINUE = XFIL(5) + FREQ
RFIL(LL) = (XFIL(1)**2 + XFIL(2)**2)**.5
THFIL(LL) = RFIL(LL)*1000.
IF(XFIL(2).LT.0.) THFIL(LL) = THFIL(LL) - PI
1  RERROR(LL) = ((XS(LL) - RFIL(LL))*COS(THFIL(LL))**2 + (YS(LL) -
XFIL(LL))*SIN(THFIL(LL))*COS(THFIL(LL))**2)**.5
XTAR(LL) = RFIL(LL)*SIN(THFIL(LL))
XMC(1,LL) = XTAR(LL)/1000. + XMC(1,LL)
XMC(2,LL) = YTAR(LL)/1000. + XMC(2,LL)
XVAR(1,LL) = (XTAR(LL)/1000.)**2 + XVAR(1,LL)
XVAR(2,LL) = (YTAR(LL)/1000.)**2 + XVAR(2,LL)
XYVAR(1,LL) = (XTAR(LL)/1000.)*(YTAR(LL)/1000.) + XYVAR(1,LL)
XYVAR(2,LL) = XFIL(3)*XFIL(4) + XYVAR(2,LL)
THFIL(LL) = THFIL(LL) - THFIL(LL)
IF(THFIL(LL).GT.-180.) THFIL(LL) = THFIL(LL) - 360.
IF(THFIL(LL).LT.-180.) THFIL(LL) = THFIL(LL) + 360.
BERROR(4) = XPRED(4)*RAD
XPRED(4) = XFIL(4)*RAD
XFIL(4) = 1090
WRITE(6,1090) XPR,XEX,V,S,THTA,FREQ
WRITE(6,1092) RFIL(LL),THFIL(LL),R(1,LL),BR(1,LL)
1092 = T(LL) + VS*DELTAT
XEX = XFIL(4)/RAD
CCNTINUE = XFIL(4)*RAD
XFIL(4) = 1081
WRITE(6,1081) IRE
WRITE(6,1083) XFIL,XTAR(NUM),YTAR(NUM)
WRITE(6,1083) Y,-5,1
CALL PLOTIP(X,INT,2,2)
CALL PLOTIP(XS,YS,XTAR,NUM,3)
CALL PLOTIP(XTAR,SECTION IS PUT HERE
      GAIN
RESIDUAL PLOT SECTION IS PUT HERE
WRITE(6,333) AVRESA,STDRA1,STDRA2

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```

CALL PLOTP(T,RSIDA,-NUM,1)
CALL PLOTP(T,VAA,NUM,3)
WRITE(6,334) STF11,STF12
CALL PLOTP(T,AVRFF1,STFNUM,1)
WRITE(6,335) AVRFF2,STF21,STF22
CALL PLOTP(T,RSIDF2,NUM,3)
CCONTINUE
201 AVEVA = AVEVA/(JJQQ*NUM)
AVEVA = (VARVA/(JJQQ*NUM))**.5
WRITE(6,62) AVEVA,VARVA
WRITE(6,701) JJQQ
DO 202 J=1,NUM
XMC(J,LL) = XMC(J,LL)/(JJQQ)
XVAR(J,LL) = (XVAR(J,LL)) - JJQQ*XMC(J,LL)**2)/(JJQQ-1)
XSTD(J,LL) = X(XYVAR(J,LL)) - JJQQ*XMC(J,LL)*XMC(2,LL)/(JJQQ-1)
XYVAR(J,LL) = XYVAR(J,LL)/(XSTD(J,LL))
XYVAR(2,LL) = XYVAR(1,LL)/(XSTD(1,LL))
XYVXY(LL) = XYVAR(2,LL) - JJQQ*XMC(3,LL)*XMC(4,LL)/(JJQQ-1)
XMC(5,LL) = XMC(5,LL)/FREQ
XAY = 2.*ATAN(XAY)/XVAR(1,LL) - XVAR(2,LL)
AAL1 = AAL1*RAD/2 + XVAR(2,LL)
XYA(LL) = X(XVAR(1,LL)/SIN(AAL1))
SIG1 = XYVAR(1,LL)/SIG2
SNEWXS = SQR1(SNEWXV)
SNEWVY = SQR1(SNEWYV)
SNEWYS = SQR1(SNEWYV)
WRITE(6,702) LL,XMC(J,LL),J,LL,J=1,5)
WRITE(6,703) LL,XVAR(J,LL),J=1,5)
WRITE(6,704) XSTD(J,LL),J=1,5)
WRITE(6,706) XYA(LL),RVXVY(LL)
WRITE(6,707) XAY,SNEWXS,SNEWVY
WRITE(6,709) SNEWYS,SNEWXS*1000,
XMN(LL) = XMC(1,LL)*1000.
YMN(LL) = XMC(2,LL)*1000.
WRITE(6,700) JJQQ
XINT(1) = 0
XINT(2) = 1000./JJQQ
XINT(3) = 1000./JJQQ
XINT(4) = 1000./JJQQ
XINT(5) = 1000./JJQQ
CALL PLOTP(XINT,NUM,2)
CALL PLOTP(XS,NUM,3)

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204 CALL PPYF(NUM)
CONTINUE
END
SUBROUTINE INIT11(AVS,FO,XFIL,XIN,YIN,SA,NUM)
DIMENSION Z(100),YS(100)
DIMENSION XS(100),Y(100)
COMMON XS,YS,X,Z,VP,W,ATHALF,ADELF,ADELTH,ATHETH,KJ,NBU
FORMAT(10,2,5X,'ADELF',F10.2,5X,'ATHETH',F10.2,5X,'Z(1,2)',
1101 F10.2,5X,'TIME OF ESTIMATE',F10.2,10X,'BEARING',F10.2,10X,'RANGE',
1 F10.2,10X,'TRUE RANGE',F10.2,10X,'ADELT*VQ/DTH)*(SIN(THES-AVET))**2
RCP(VQ,DTH,THES,AVET) = W*(ADELT*VQ/DTH)*SIN(THES-AVET)**2
XCPC(VQ,DTH,THES,AVET) = W*(ADELT*VQ/DTH)*SIN(THES-AVET)/2.
RNG(VQ,DTH,THES,AVET) = FAVE*(1.+VQ/DTH)*SIN(THES-AVET)
RAD = 57.29578
KJ = KJ + 1
90 ADELTH = Z(KJ,1) - Z(1,1)
ADELF = Z(KJ,2) - Z(1,2)
IF(KJ.EQ.NUM) GO TO 900
IF(ABS(ADELTH).LT.3.*SA) GO TO 90
CONTINUE
IF(ADELTH.LT.0.) W = - 1.0
ADELT = T(KJ) - T(1)
ATHALF = T(KJ)/2. + T(1)/2.
ADELF = Z(KJ,2) - Z(1,2)
IF(ADELTH.EQ.0.) GO TO 90
ATHETH = 90.
ATHETH = 1./KJ + ATHETH
ATHETH(6,667) = Z(KJ,2),Z(1,2),ADELF,ATHETH
92 WRITE(6,667) T(KJ),T(1),Z(KJ,2),Z(1,2),ADELF,ATHETH
ATHRUE = ((XS(KJ)+XS(1))/2.)*SIN(THES-AVET) + YS(1)/2.)*SIN(THES-AVET)**.5
95 ASINE = -ADELF*VP/(ADELF+RAD)*Z(KJ,2)*AVS
IF(ABS(ASINE).LE.1.) GO TO 91
INITIAL SPEED TOO SLOW. INCREASE BY TWO KNOTS.
AVS = AVS + 2.*.563
GO TO 95
91 HDT = RAD*ASINE
HDT = 180. + ATHETH - HDT
IF(Z(1,2).GT.FO) HDING = ATHETH + HDT
IF(Z(1,2).LT.FO) HDING = HDING - 360.
IF(HDING.GT.180.) HDING = HDING + 360.
IF(HDING.LT.-180.) HDING = HDING + 360.
ESTIMATE OF REST FREQUENCY.
FAVE = 0.
DO 130 JJ=1,KJ

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130 FAVE = Z(JJ,2)/KJ + FAVE
DTHI = ADELTH/RAD
HDII = HDING/RAD
ATHI = ATHETH/RAD
FO = FOCPIAVS,HDII,ATHI,
ARHALF = RNG(AVS,DTHI,HDII,ATHI)/1000.
XFIL(1) = RCP(AVS,DTHI,HDII,ATHI)/1000.
XFIL(2) = XCP(AVS,DTHI,HDII,ATHI)/1000.
IF(NBU.EQ.1) GO TO 944
C C C
ONE DIFA AND AT LEAST ONE LOFA.
F2AVE = Z(KJ,3)/2 + Z(1,3)/2.
941 COTEM = (VP/AVS)*(FO/F2AVE - 1.)
C C C
ADJUST FO IF ABS(COTEM) IS GREATER THAN 1.
IF(COTEM.LT.-1.) FO = FO + .05
IF(COTEM.GT.+1.) FO = FO - .05
IF(ABS(COTEM).GT.1.) GO TO 941
ANTEM = ARCOS(COTEM)*RAD
C C C
TWO POSSIBLE BEARINGS TO THE SECOND BUOY.
TH2(1) = HDING - ANTEM
TH2(2) = HDING + ANTEM
D2 = ((X(2) - X(1))*2 + (Y(2) - Y(1))*2)**.5
T2 = ATAN((Y(2) - Y(1))/(X(2) - X(1)))*RAD
IF((X(2) - X(1)).GE.0.) GO TO 942
IF((Y(2) - Y(1)).LE.0.) T2 = T2 + 180.
IF((Y(2) - Y(1)).GE.0.) T2 = T2 + 180.
T2DF = (T2 - 2*ATHETH)/RAD
942 DC 943 KA=1,TH2(KA)/RAD
T2DF = (T2 - 2*TH2(KA))/RAD
TDF = 1./TAN(T2DF) - 1./TAN(T2DF)
ARX(KA) = 1./D2/SIN(ARHALF)
IF(ABS(ARX(1)) - ARHALF).LT.ABS(ARX(2) - ARHALF)) ARHALF = ARX(1)
IF(ABS(ARX(2)) - ARHALF).LT.ABS(ARX(1) - ARHALF)) ARHALF = ARX(2)
XFIL(1) = ARHALF*ABS(ASINE)/1000.
XFIL(2) = SQRT((ARHALF/1000.)*2 - XFIL(1)**2)
IF(Z(1,2).GT.FO) XFIL(2) = -XFIL(2)
944 CONTINUE = AVS
XFIL(3) = HDING
XFIL(4) = FO
XFIL(5) = ARHALF*COS(ATHI)
XVIN = ARHALF*SIN(ATHI)
WRITE(6,1101) ATHALF,ATHETH,ARHALF,ARTRUE
C

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```

RETURN
ENDROUTINE INIT20(AVS,FO,XFIL,XIN,YIN)
SUBROUTINE Z(100,6),XFIL(5),X(6),Y(6),XS(100),YS(100)
COMMON XS,X,Y,Z,I,VP,W,ATHALF,ADELTH,ATHEH,KJ,NBU
RAD = 57.29578
Q2 = ((X(2)**2 + (Y(2) - Y(1))**2)**0.5
T2 = ATAN((X(2) - X(1))/(Y(2) - Y(1)))*RAD
IF((X(2) - X(1))*.GE.0.) GO TO 898
IF((Y(2) - Y(1))*.GE.0.) T2 = T2 + 180.
IF((Y(2) - Y(1))*.LT.0.) T2 = T2 - 180.
DFF = 1./TAN((T2 - Z(1,1))/RAD) - 1./TAN((T2 - Z(1,3))/RAD)
ADHALF = D2/(SIN((T2 - Z(1,1))/RAD)*DFF}
ADELTH = Z(1,4) - Z(1,2)
ATHEH = Z(1,3)/2. + Z(1,1)/2.
ASINE = -ADELF*VP/(2.*SIN(ADELTH/(2.*RAD))*Z(1,2)*AVS)
IF(ABS(ASINE).LE.1.) GO TO 897
AVS = AVS + 2.*.563
GO TO 896
HCT = RAD*ASIN(ASINE)
FFAVE = Z(1,2)/2. + Z(1,4)/2. + ATHEH - HDT
IF(FFAVE*.GE.FO) HDING = 180. + ATHEH + HDT
IF(FFAVE*.LT.FO) HDING = ATHEH + HDT
IF(HDING*.GT.+180.) HDING = HDING - 360.
IF(HDING*.LT.-180.) HDING = HDING + 360.
SUM = 0.0
DC 895 KN=2,4,2
FOT = (Z(1,KN)/VP)*(VP + AVS*COS((HDING - Z(1,KN-1))/RAD))
SUM = SUM + FOT
FUC = SUM/2.
IF(ABS(Z(1,1) + 90. - HDING).GT.90.) W = -1.0
XIN = ARHALF*COS(Z(1,1)/RAD)
XYIN = ARHALF*SIN(Z(1,1)/RAD)
XVIN = XIN/1000.
YVIN = YIN/1000.
XFIL(1) = ABS(ARHALF*SIN((HDING - Z(1,1))/RAD))/1000.
XFIL(2) = ((ARHALF/1000.)*2 - XFIL(1)**2)**0.5
XFIL(3) = (GT.FO) XFIL(2) = -XFIL(2)
XFIL(4) = AVS
XFIL(5) = HDING
XKJ = FOT
RETURN
ENDFUNCTION HEAD(DF,DT,VS,AV)
SUBROUTINE Z(100,6),XS(100),YS(100),T(100),X(6),Y(6)
COMMON XS,X,Y,Z,I,VP,W,ATHALF,ADELTH,ATHEH,KJ,NBU

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PI = 3.14159
ASINE = -VP#DF/(Z(1,2)*DT*VS)
IF(ASINE.GT.1.) ASINE = 1.
IF(ASINE.LT.-1.) ASINE = -1.
H = PI + AV - ARSIN(ASINE)
IF(Z(1,2).LT.FO) H = AV + ARSIN(ASINE)
IF(H.GT.PI) H = H - 2.*PI
IF(H.LT.-PI) H = H + 2.*PI
HEAD = H
RETURN
END
SUBROUTINE PPYF(NUM)
DIMENSION XPT(20), YPT(20)
DIMENSION XS(100), YS(100), X(6), Y(6), Z(100,6), T(100)
COMMON XS, YS, X, Y, Z, T, VPM, W, ATHALF, ADELTH, ATHETH, KJ, NBU
COMMON/BLK1/XMN(20), M1
REAL XTITLE(1)/MONT/, ECA, RLO, SIMU, LATI,
1. ON - , T(200), RITS, CHAN, GFI, ALTER, OUT,
REAL GEOINCREMENT IS DEX AND DEY
REAL AXIS = 1333.
DEY = 1333.
DO 1 X(1) = 0.
X(1) = 0.
X(2) = -1.
X(3) = -1.
X(4) = 1.
CALL SYMBOL(0,0,0,35,MIT,0,0,20)
CALL SYMBOL(5,6,0,35,MIT,0,0,20)
CALL PLQIS(0,-4,0,0,XTITLE,-1,9,0,90,-5000,DEY)
CALL AXIS(0,-2,25,0,0,XTITLE,-1,9,0,90,-5000,DEY)
CALL AXIS(-2,25,0,0,XTITLE,-1,6,0,0,-2500,DEX)
CALL LINE(X,Y,M1,1,4)
CALL LINE(X,Y,M1,1,4)
DRAW KK=1, XS(KK)/DEX
DO 2 XPT(1) = XS(KK)
XPT(1) = YS(KK)/DEY
CONTINUE
CALL LINE(XPT,YPT,NUM,1,3)
CALL LINE(XPT,YPT,NUM,1,3)
DO 1 J=1, NUM

```

C

C


```

C
1
XPT(J) = XMN(J)/DEX
YPT(J) = YMN(J)/DEY
CONTINUE
DRAW LINE(INTER OUTPUT NUM,1,-5)
CALL LINE(XPT,YPT,NUM,1,-5)
CALL LINE(XPT,YPT,NUM,1,-5)
CALL SYMBOL(.5,4.,.14,TITLE,0.,36)
CALL SYMBOL(.5,4.,.14,TITLE,0.,36)
CALL PLOT(1-3.50,6.5,-3)
CALL PLOT
RETURN
END
//LINK.SYSPRINT DD DUMMY
//GO.SYSIN DD *
65473195 6521475
3
1000. 6000. 8.0 -70.
-500. -4000. 6.0 65.0
-750. 6000. 9.0 -65.0
200

```


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